

EE 330

Lecture 17

MOSFET Modeling

Spring 2024 Exam Schedule

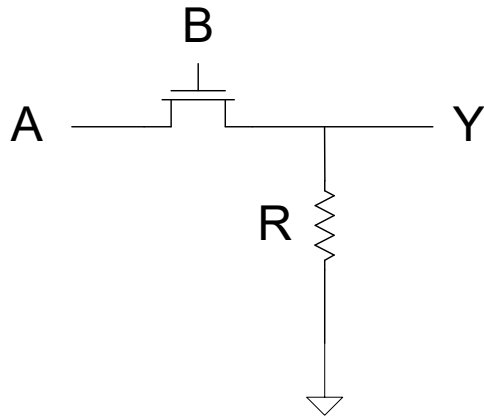
Exam 1 Friday Feb 16

Exam 2 Friday March 8

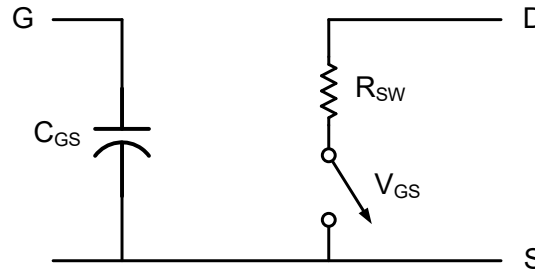
Exam 3 Friday April 19

Final Exam Tuesday May 7 7:30 AM - 9:30 AM

Limitations of Existing MOSFET Models



What is Y when $A=B=V_{DD}$

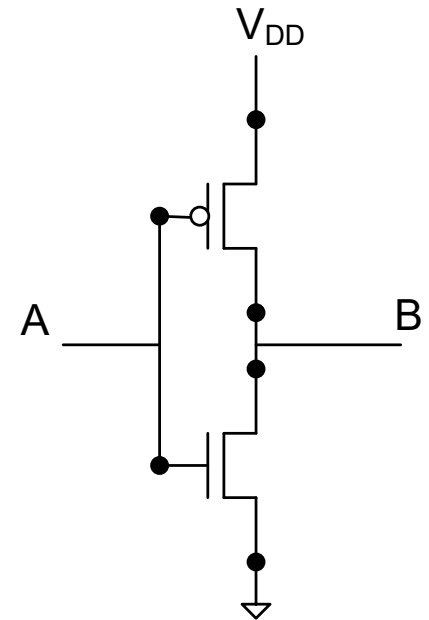


For minimum-sized devices in a 0.5u process with $V_{DD}=5V$

$$C_{GS} \cong 1.5fF$$

$$R_{sw} \cong \left. \begin{array}{l} 2K\Omega \text{ n-channel} \\ 6K\Omega \text{ p-channel} \end{array} \right\}$$

What is R_{sw} if MOSFET is not minimum sized?

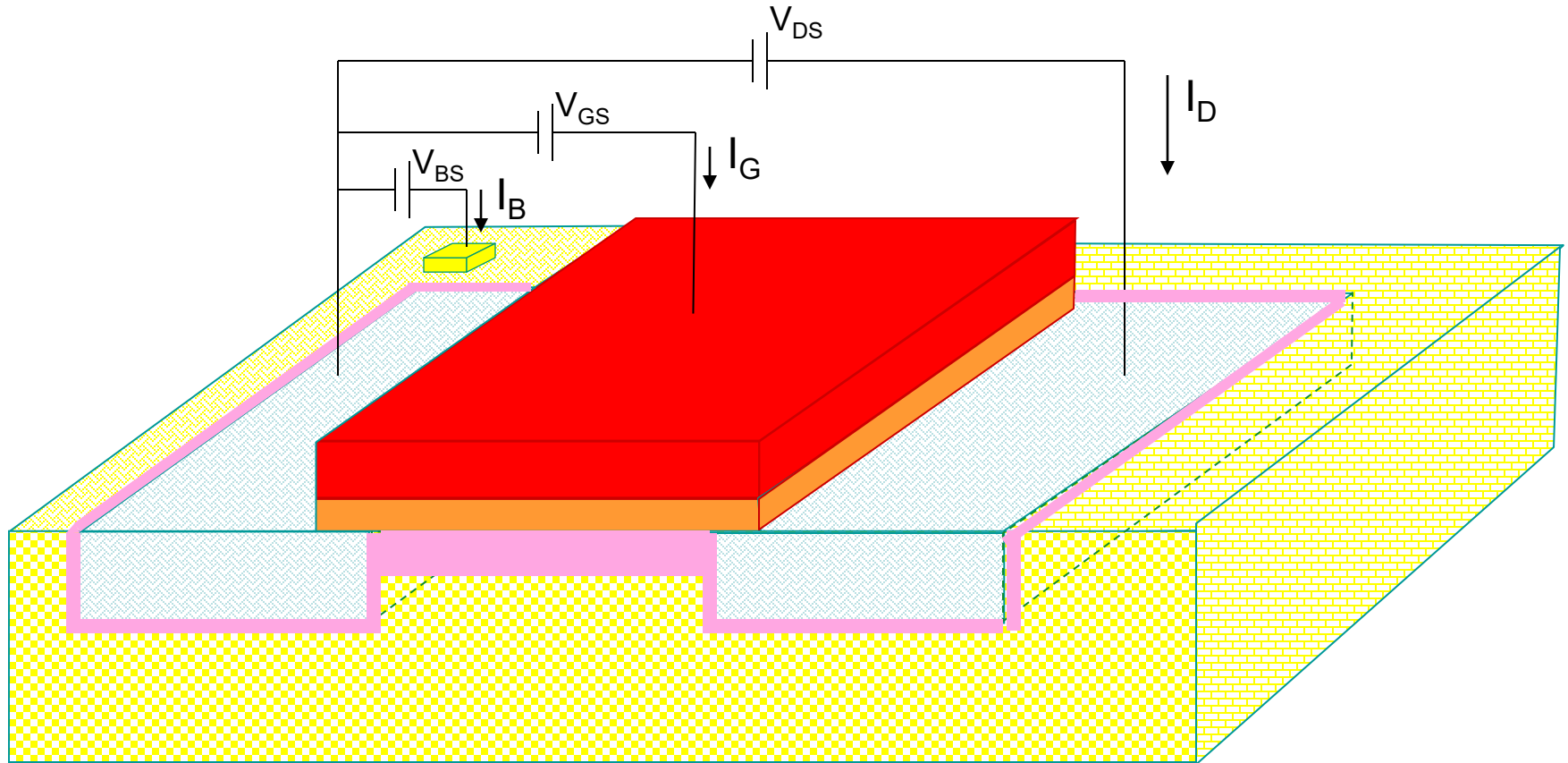


What is power dissipation if A is stuck at an intermediate voltage?

Better Model of MOSFET is Needed!

Review from last lecture

n-Channel MOSFET Operation and Model

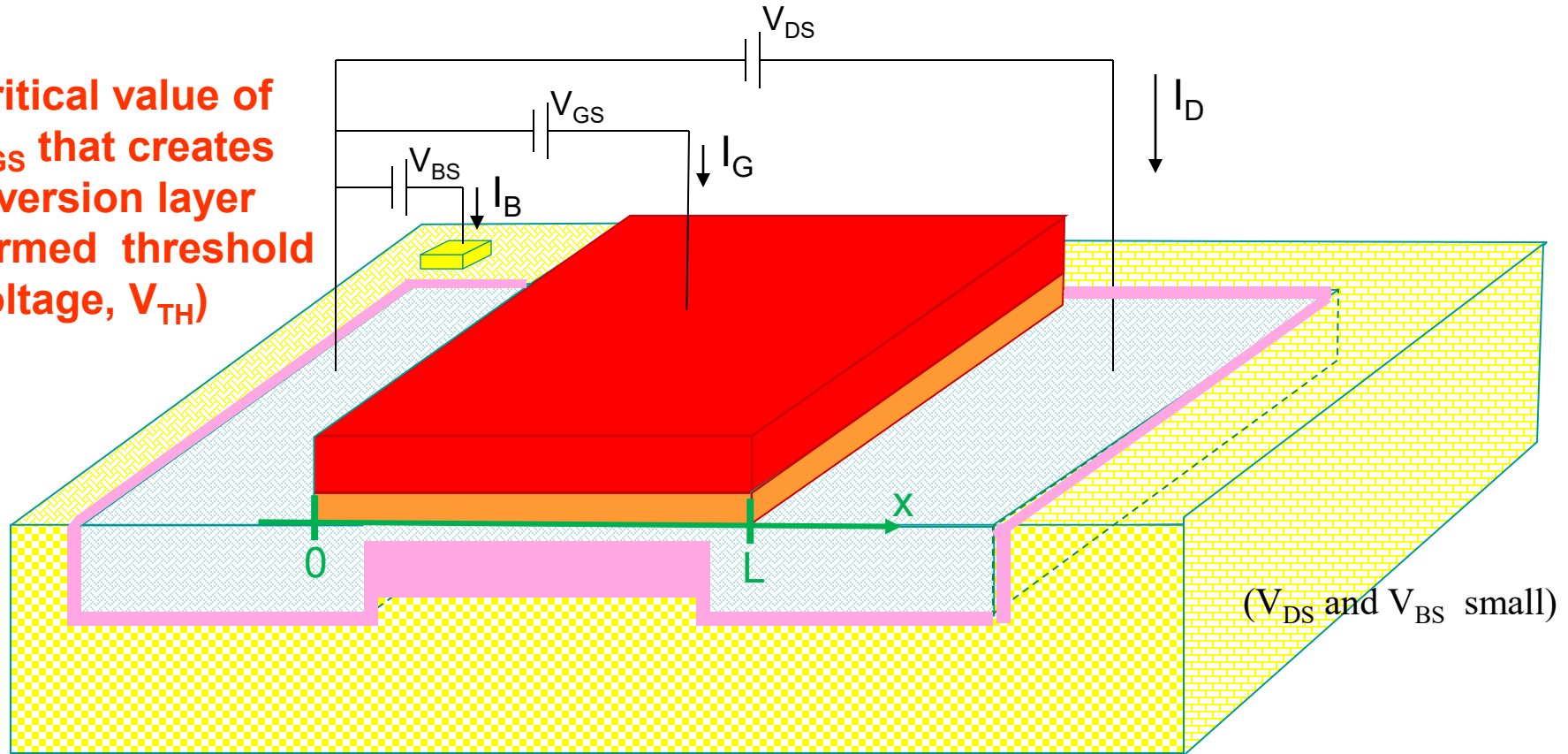


$$\begin{aligned} I_D &= 0 \\ I_G &= 0 \\ I_B &= 0 \end{aligned}$$

Model in Cutoff Region

n-Channel MOSFET Operation and Model

Critical value of V_{GS} that creates inversion layer termed threshold voltage, V_{TH}



Increase V_{GS} more

Inversion layer forms in channel

Inversion layer will support current flow from D to S

Channel behaves as thin-film resistor

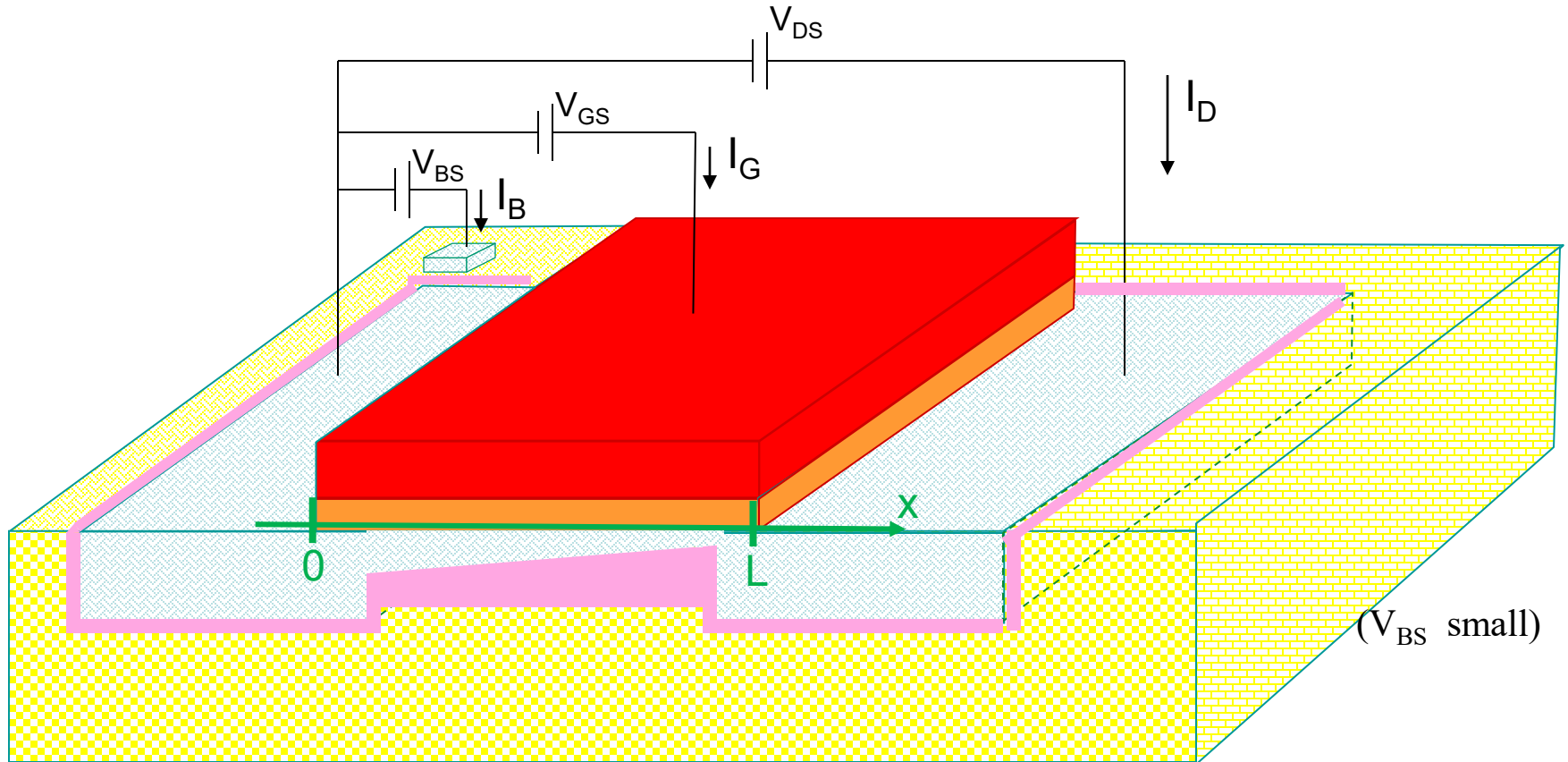
$$I_D R_{CH} = V_{DS}$$

$$I_G = 0$$

$$I_B = 0$$

Review from last lecture

n-Channel MOSFET Operation and Model



Increase V_{DS} and $V_{GS} > V_{TH}$

$V_{GC}(x)$ changes with x for larger V_{DS}

Inversion layer thins near drain

I_D no longer linearly dependent upon V_{DS}

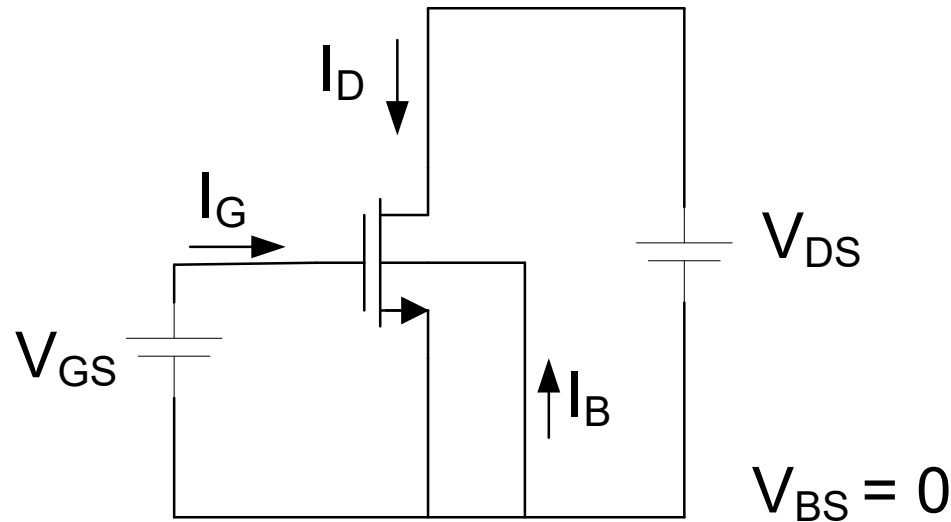
Still termed “ohmic” or “triode” region of operation

$$I_D = ?$$

$$I_G = 0$$

$$I_B = 0$$

Triode Region of Operation



For V_{DS} larger

~~$$R_{CH} = \frac{L}{W (V_{GS} - V_{TH}) \mu C_{OX}}$$~~

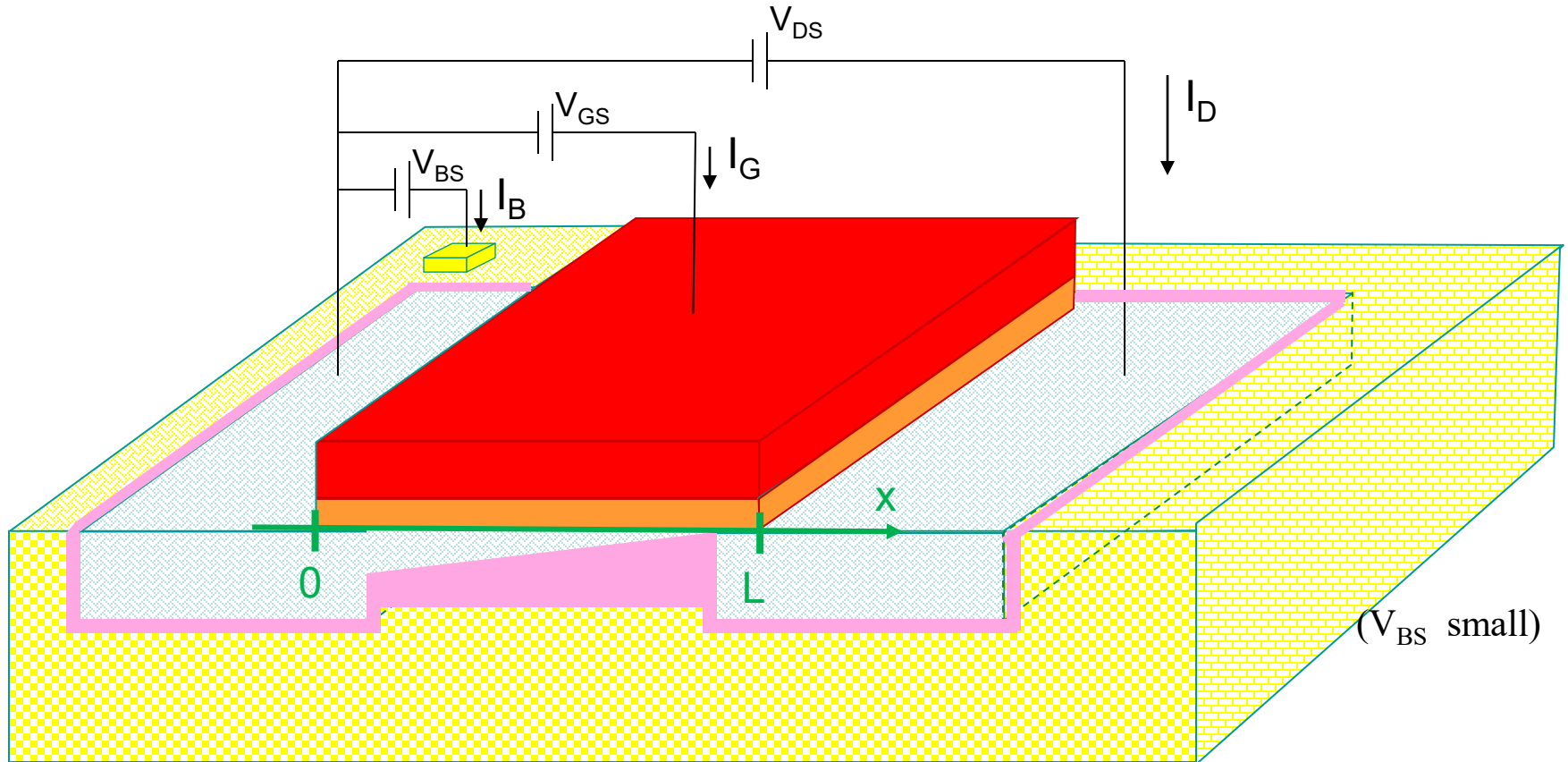
$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

$$I_G = I_B = 0$$

Model in Triode Region

Review from last lecture

n-Channel MOSFET Operation and Model



Increase V_{DS} even more (beyond $V_{GS} - V_{TH}$)

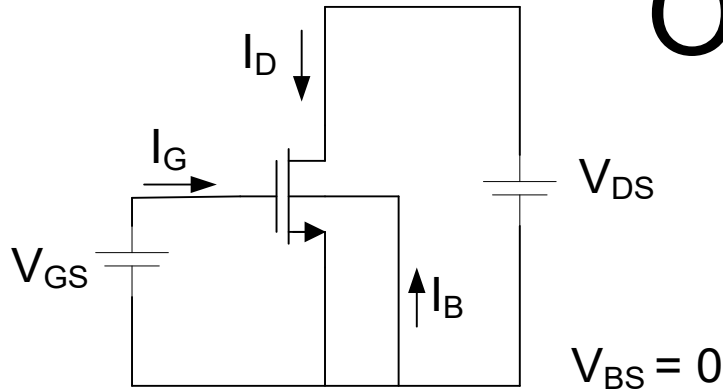
Nothing much changes !!

Termed “saturation” region of operation

$$\begin{aligned} I_D &=? \\ I_G &=0 \\ I_B &=0 \end{aligned}$$

Review from last lecture

Saturation Region of Operation



For V_{DS} in Saturation

$$I_D = \frac{\mu C_{OX} W}{2L} (V_{GS} - V_{TH})^2$$

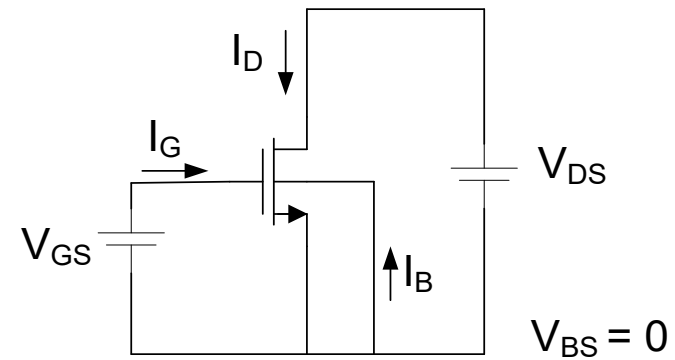
$$I_G = I_B = 0$$

Model in Saturation Region

Model Summary

n-channel MOSFET

Notation: Don't confuse V_{TH} with $V_t = kT/q$



$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} & \text{Cutoff} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T & V_{DS} < V_{GS} - V_{TH} & \text{Triode} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 & V_{GS} \geq V_T & V_{DS} \geq V_{GS} - V_{TH} & \text{Saturation} \end{cases}$$

$$I_G = I_B = 0$$

Model Parameters: $\{\mu, V_{TH}, C_{OX}\}$ Design Parameters: $\{W, L\}$

This is a piecewise model (not piecewise linear though)

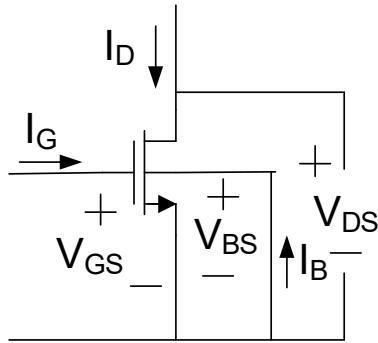
Piecewise model is continuous at transition between regions

(Deep triode special case of triode where V_{DS} is small $R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$)

Note: This is the third model we have introduced for the MOSFET

Model Summary

n-channel MOSFET



$$V_{BS} = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

$$I_G = I_B = 0$$

Observations about this model (developed for $V_{BS}=0$):

$$I_D = f_1(V_{GS}, V_{DS})$$

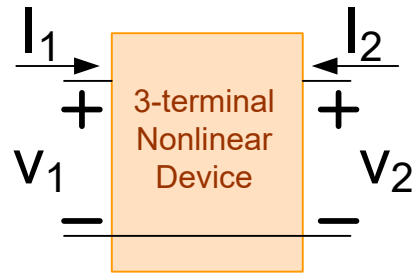
$$I_G = f_2(V_{GS}, V_{DS})$$

$$I_B = f_3(V_{GS}, V_{DS})$$

This is a nonlinear piecewise model characterized by the functions f_1 , f_2 , and f_3 where we have assumed that the port voltages V_{GS} and V_{DS} are the independent variables and the drain currents are the dependent variables

Review from last lecture

General Nonlinear Models

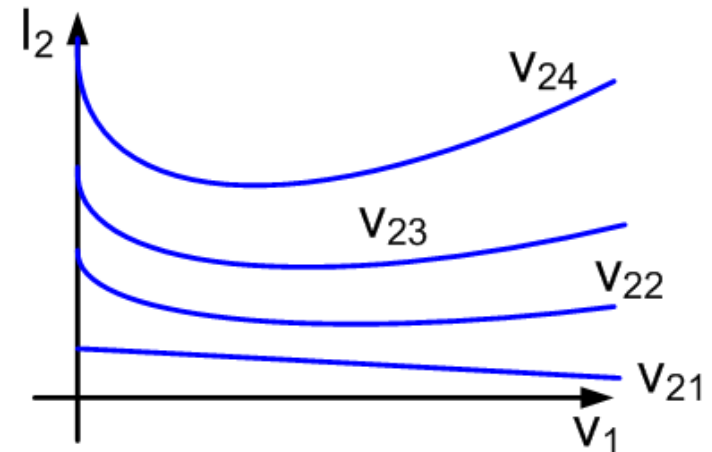
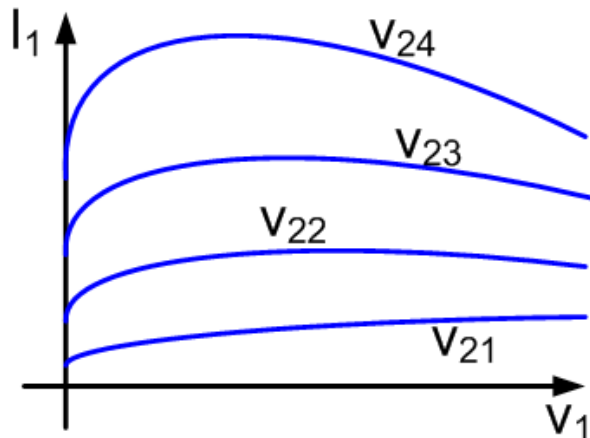


$$I_1 = f_1(V_1, V_2)$$

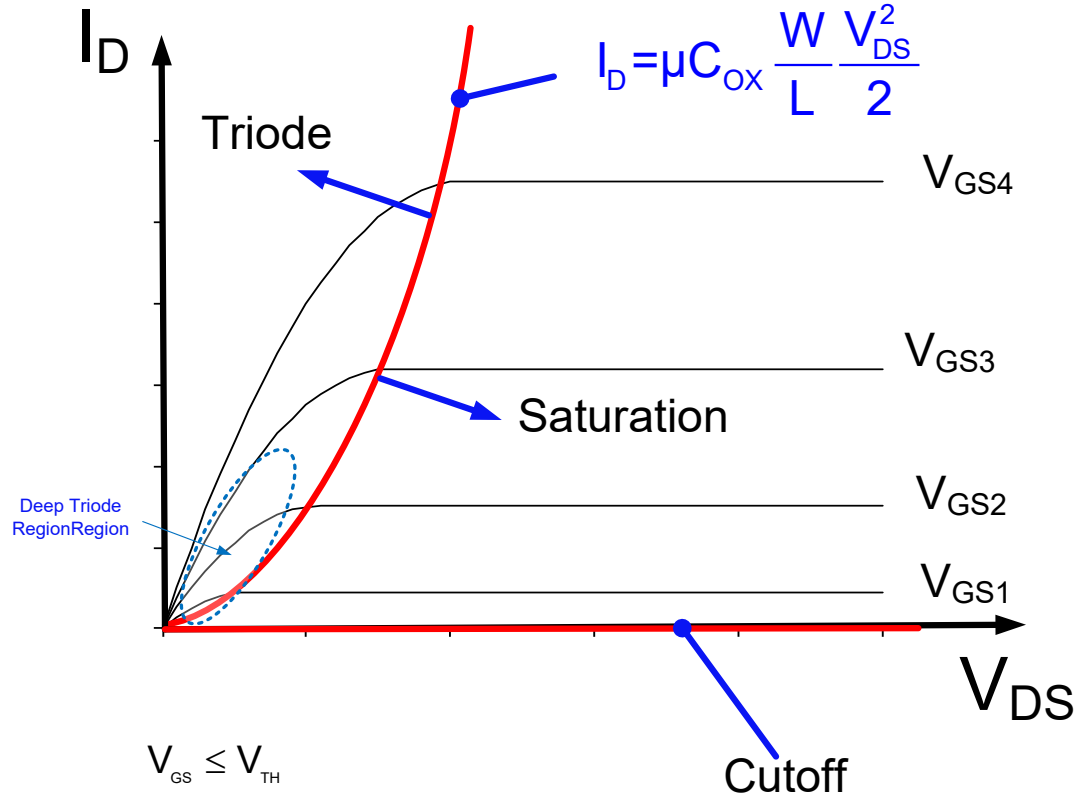
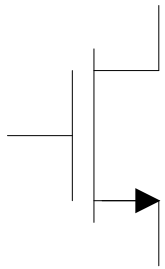
$$I_2 = f_2(V_1, V_2)$$

I_1 and I_2 are 3-dimensional relationships which are often difficult to visualize

Two-dimensional representation of 3-dimensional relationships



Graphical Representation of MOS Model

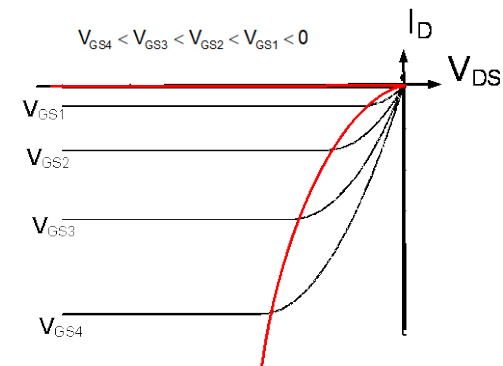
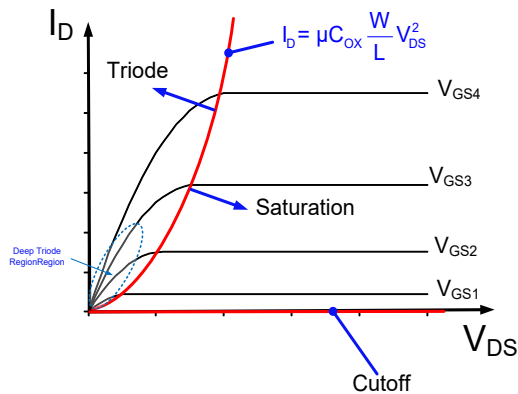
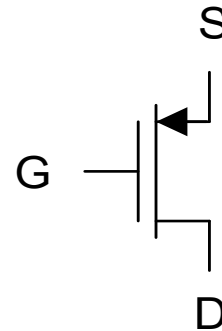
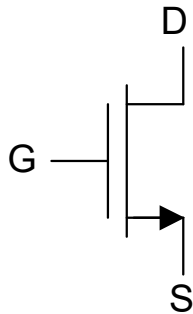


$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{ox} \frac{W}{2L} (V_{GS} - V_{TH})^2 & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

$$I_G = I_B = 0$$

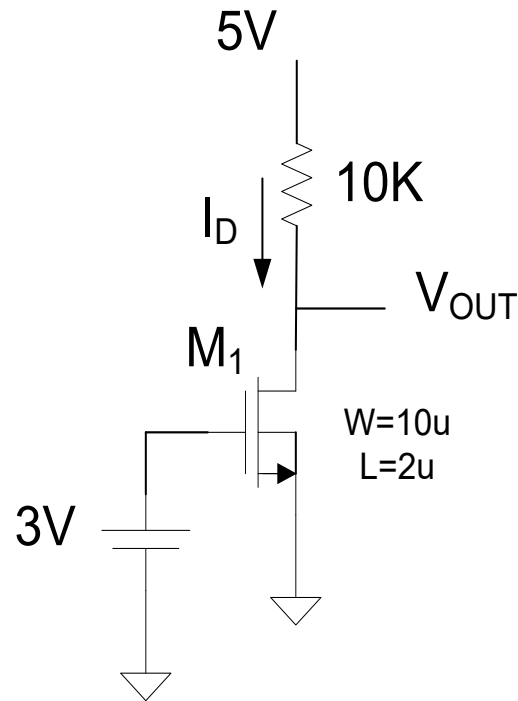
Parabola separated triode and saturation regions and corresponds to $V_{DS} = V_{GS} - V_{TH}$

PMOS and NMOS Models



- Functional form identical, sign changes and parameter values different
- Will give details about p-channel model later

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_{TH}=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

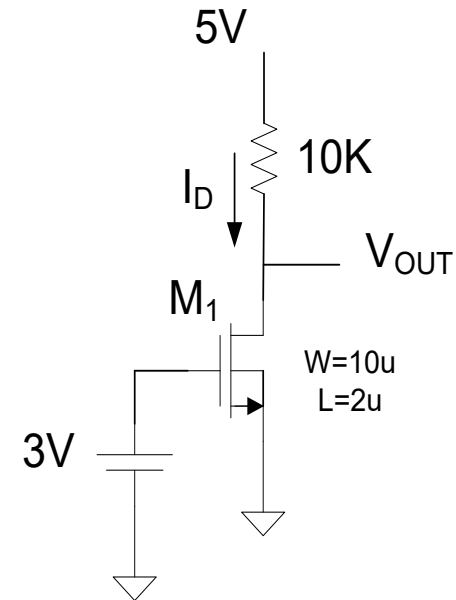


Solution:

Since $V_{GS} > V_{TH}$, M_1 is operating in either saturation or triode region

Strategy will be to guess region of operation, solve, and then verify region

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_{TH}=1V$ and $\mu C_{OX}=100\mu AV^{-2}$



Solution:

Guess M_1 in saturation

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{2L} (3 - V_{TH})^2$$

Required verification: $V_{DS} > V_{GS} - V_{TH}$

Can eliminate I_D between these 2 equations to obtain V_{OUT}

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_{TH}=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

Guess M_1 in saturation

Required verification: $V_{DS} > V_{GS} - V_T$

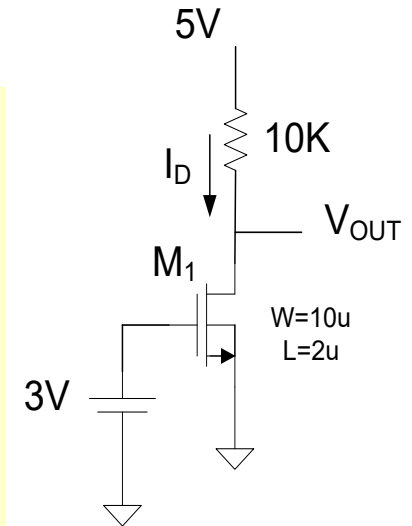
$$\left. \begin{aligned} 5V &= I_D 10K + V_{OUT} \\ I_D &= \frac{\mu C_{OX} W}{2L} (3 - V_{TH})^2 \end{aligned} \right\}$$

$$V_{OUT} = 5V - 10K \left[\frac{100\mu AV^{-2} 10\mu}{2 \cdot 2\mu} (2V)^2 \right]$$

$$V_{OUT} = -5V$$

Verification: $V_{DS} = V_{OUT}$

$-5 >? 2V - - 0$ No! So verification fails and Guess of region is invalid



Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_{TH}=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

Guess M_1 in triode

Required verification: $V_{DS} < V_{GS} - V_T$

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{L} \left(3 - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

$$V_{OUT} = 5V - 10K \left[\frac{100\mu AV^{-2} 10\mu}{2\mu} \left(2V - \frac{V_{OUT}}{2} \right) V_{OUT} \right]$$

$$V_{OUT} = 5V - \left[5 \left(2V - \frac{V_{OUT}}{2} \right) V_{OUT} \right]$$

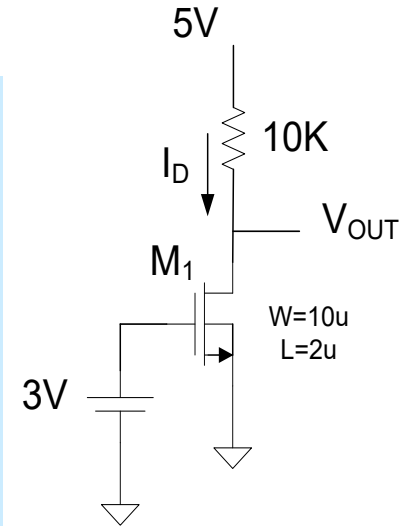
Solving for V_{OUT} , obtain

$$V_{OUT} = 0.515V$$

Verification: $V_{DS} = V_{OUT}$

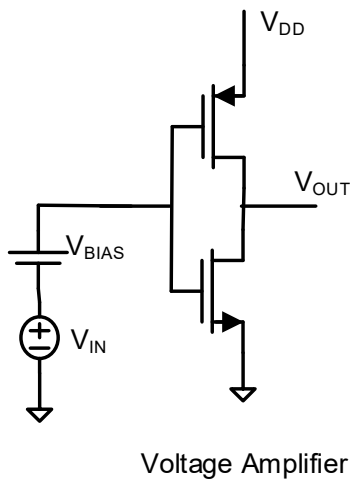
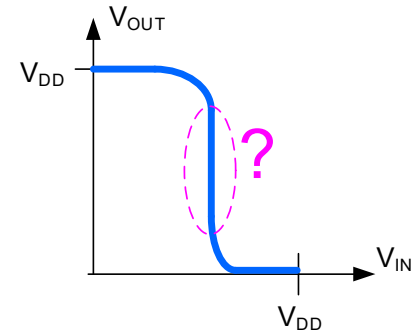
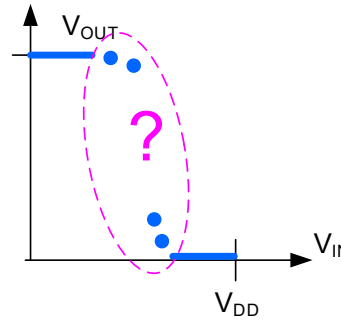
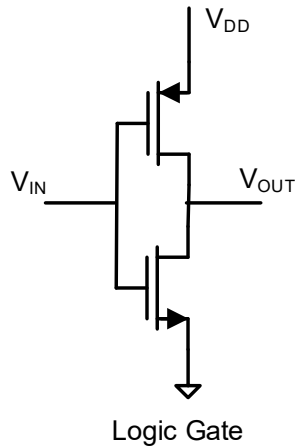
$0.515 <? 2V$ Yes!

So verification succeeds and triode region is valid



$$V_{OUT} = 0.515V$$

Limitations of Existing Models



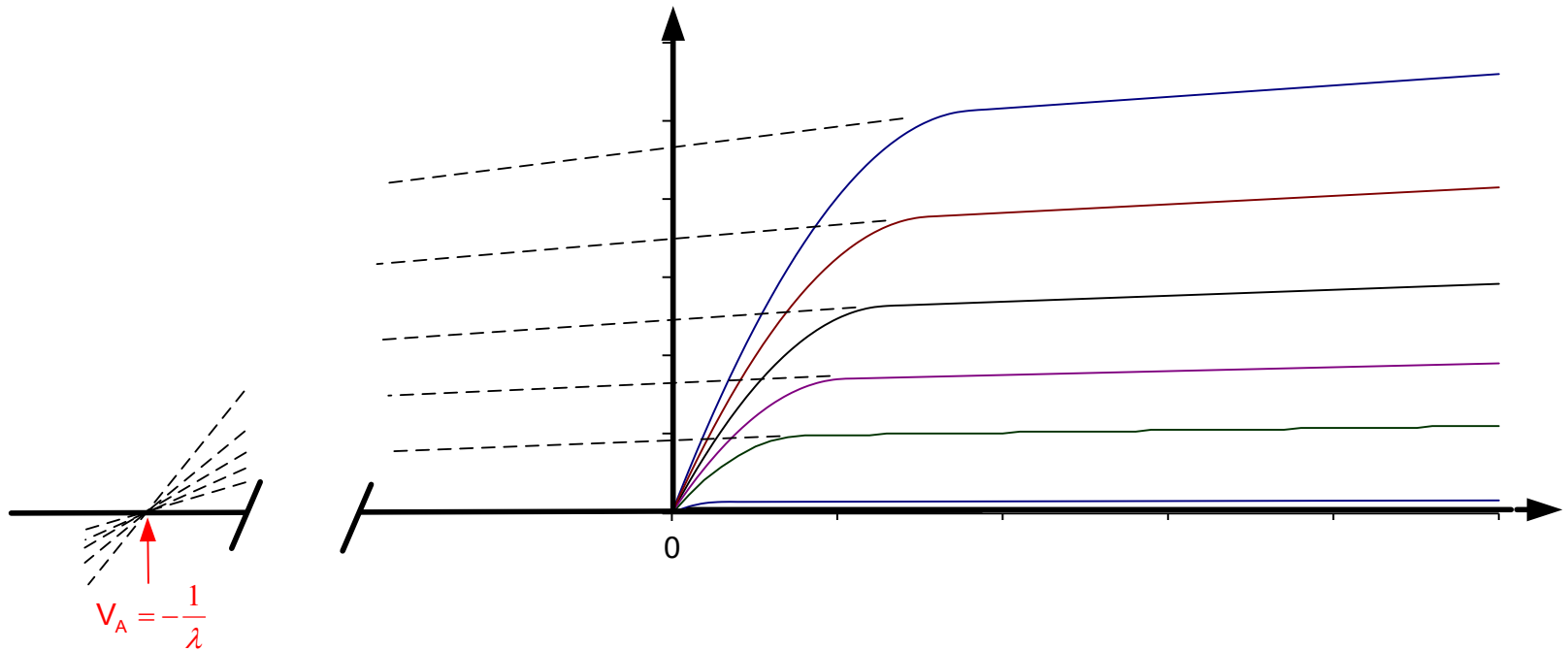
Switch-Level Models

Simple square-law Model



Voltage Gain
Input/Output Relationship ?

Model Extensions

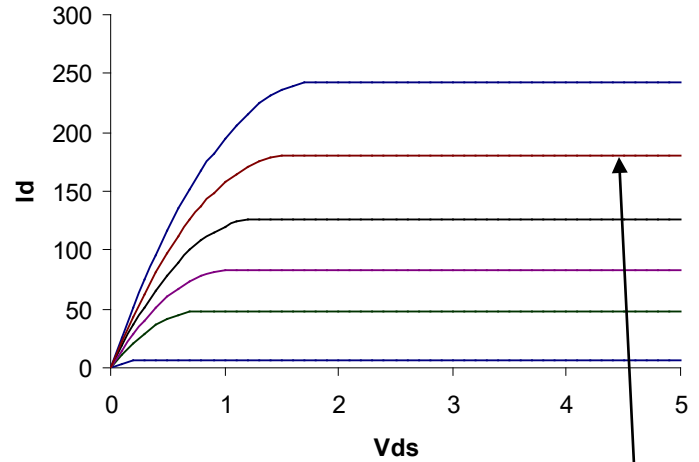


Projections intersect $-V_{DS}$ axis at same point, termed Early Voltage

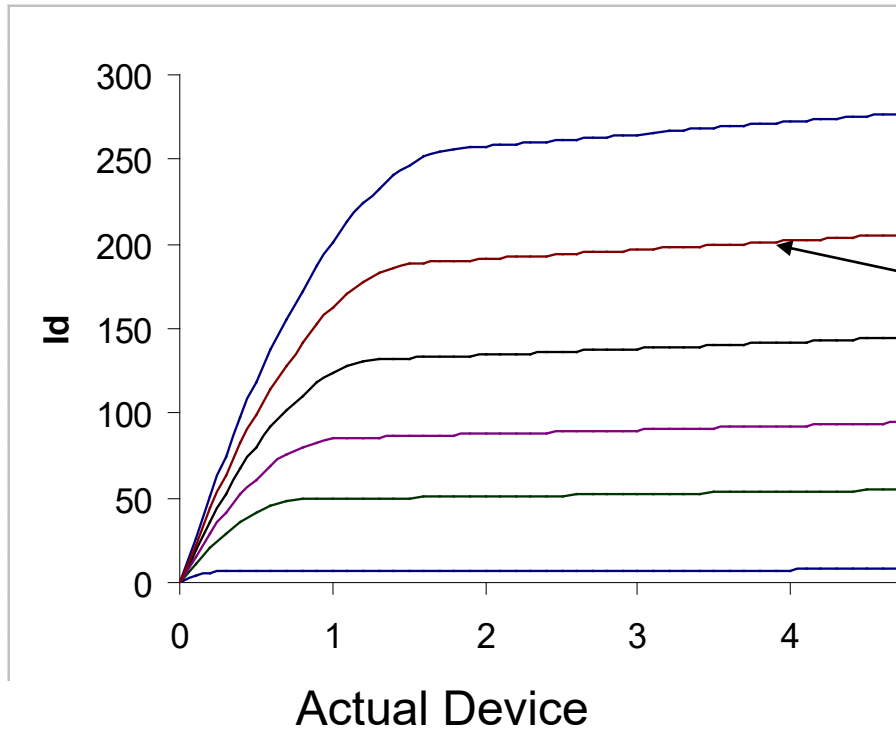
Typical values from -20V to -200V

Usually use parameter λ instead of V_A in MOS model

Model Extension



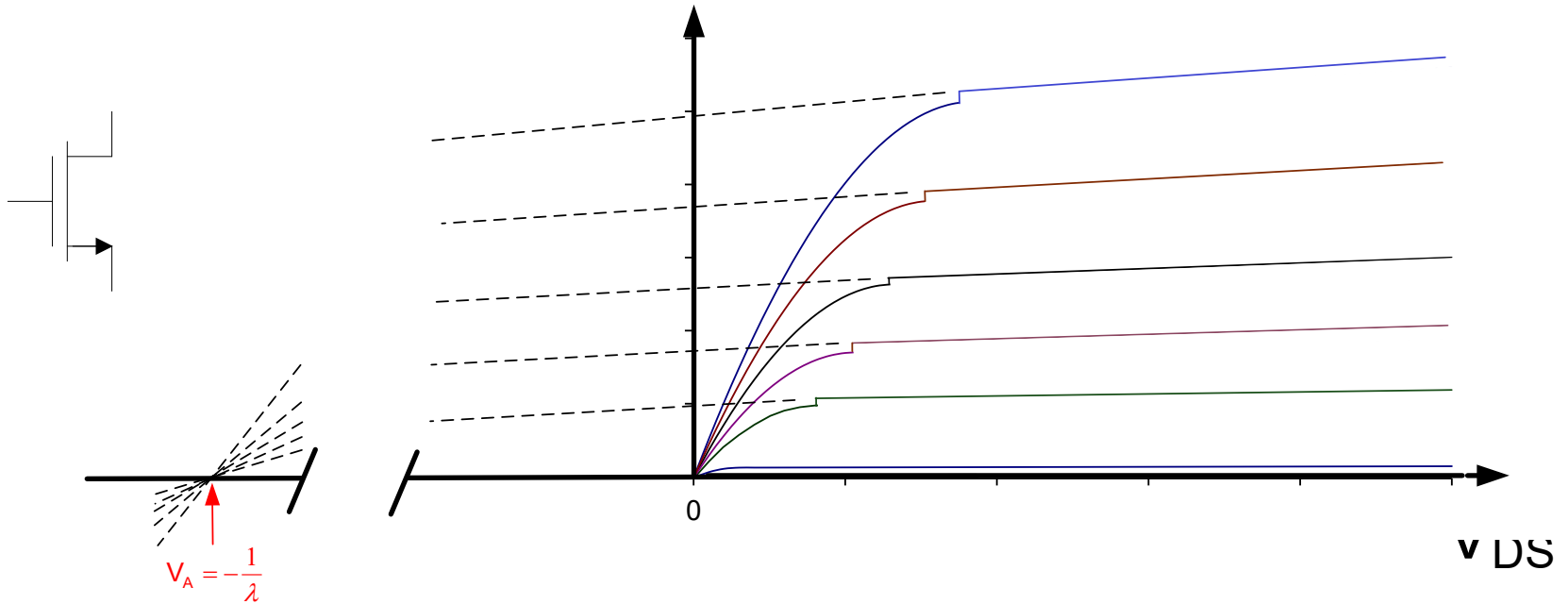
Existing Model



Actual Device

Slope is not 0

Model Extensions



$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

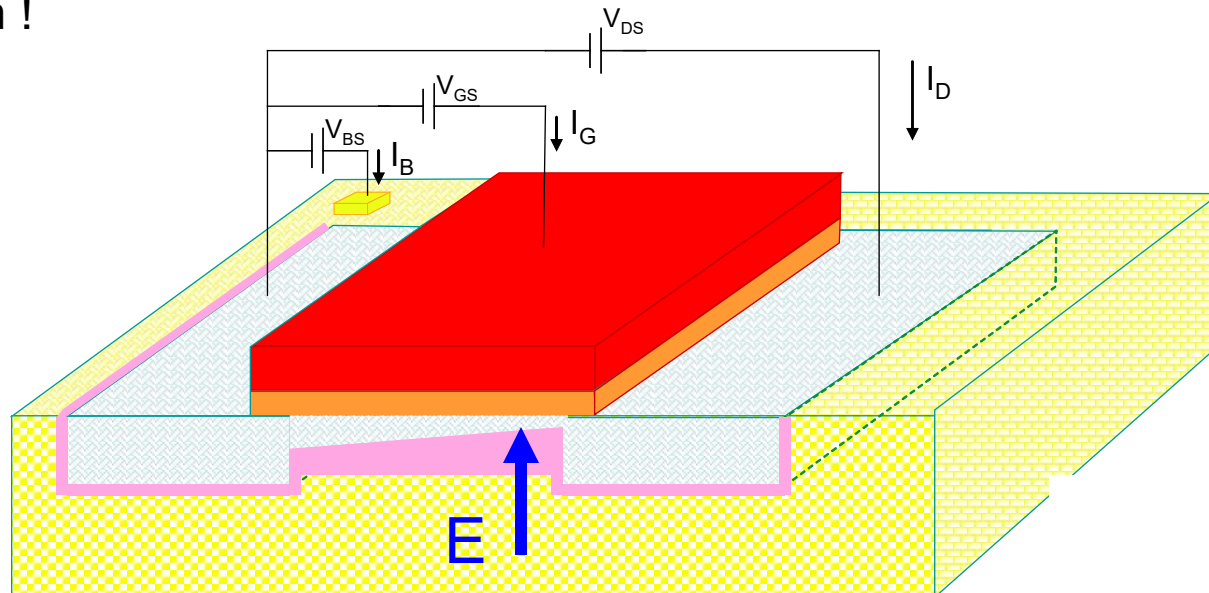
Note: This introduces small discontinuity in model at SAT/Triode transition

Further Model Extensions

Existing model does not depend upon the bulk voltage !



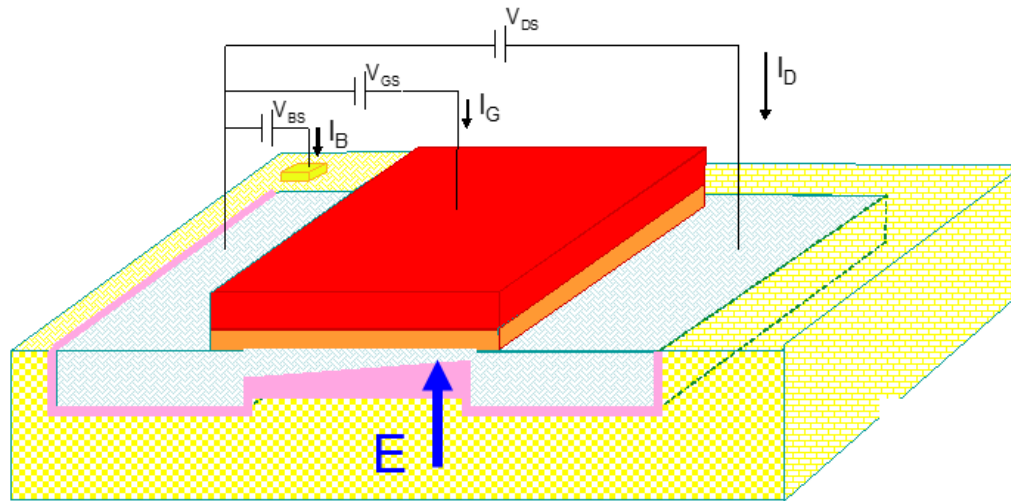
Observe that changing the bulk voltage will change the electric field in the channel region !



Further Model Extensions

Existing model does not depend upon the bulk voltage !

Observe that changing the bulk voltage will change the electric field in the channel region !



Changing the bulk voltage will change the thickness of the inversion layer

Changing the bulk voltage will change the threshold voltage of the device

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

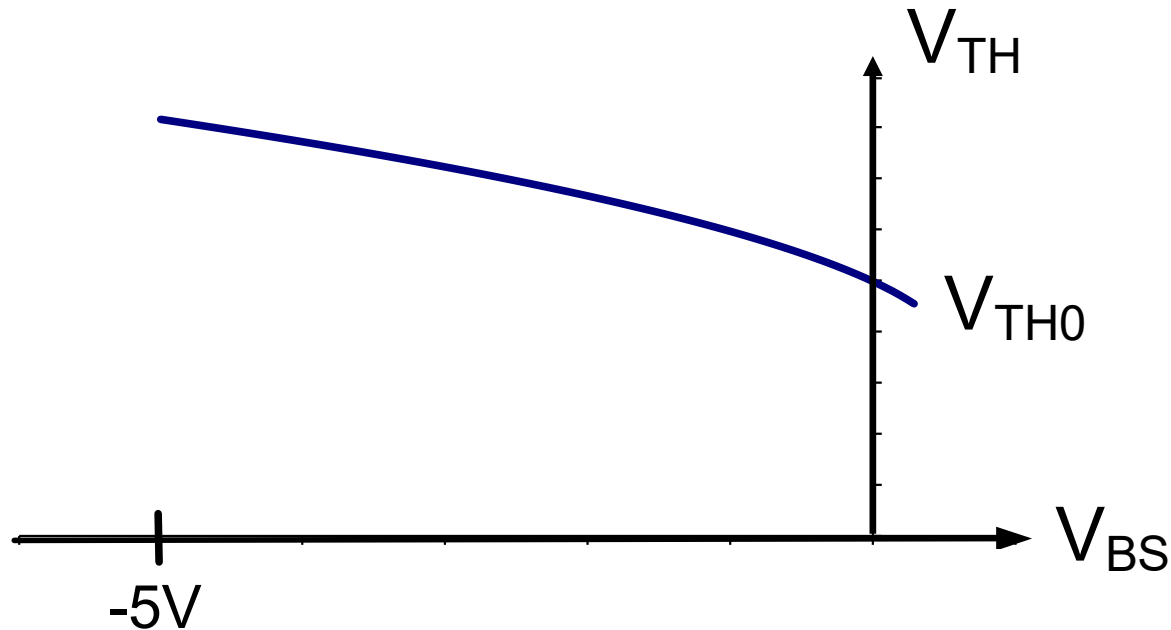
ϕ is the surface potential (some authors use symbol Φ_S)

γ is the bulk threshold

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

$$\gamma \cong 0.4V^{1/2} \quad \phi \cong 0.6V$$

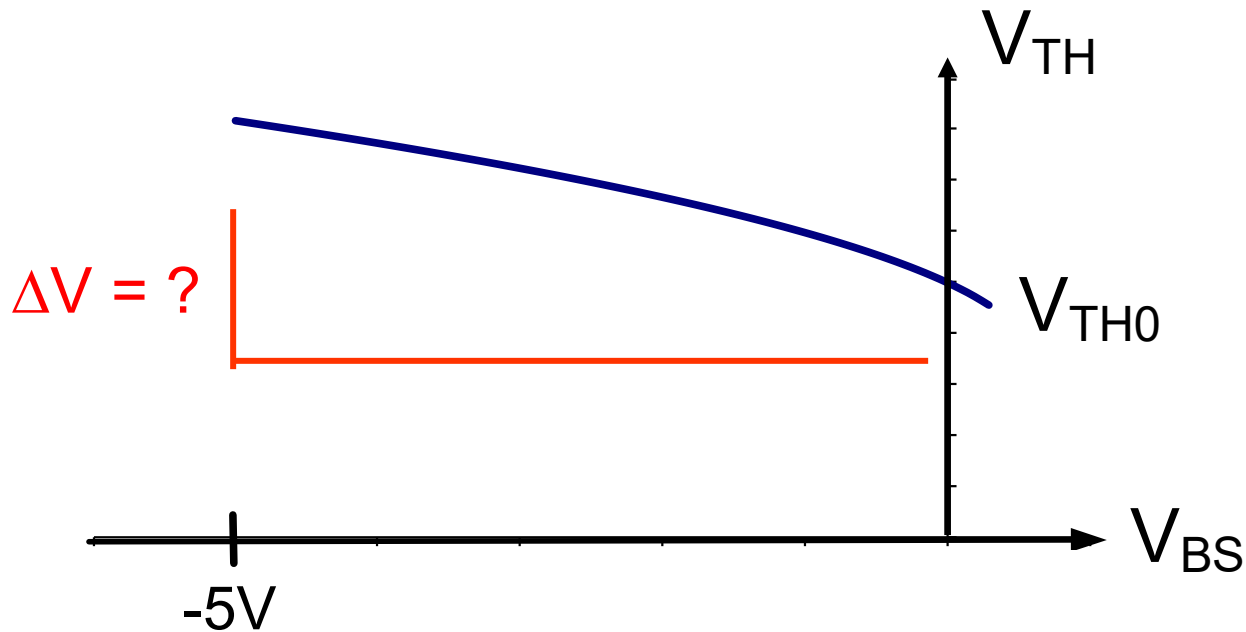


- Bulk-Diffusion Generally Reverse Biased ($V_{BS} < 0$ or at least $V_{BS} < 0.3V$) for n-channel
- Shift in threshold voltage with bulk voltage can be substantial
- Often $V_{BS} = 0$

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

$$\gamma \cong 0.4V^{1/2} \quad \phi \cong 0.6V$$



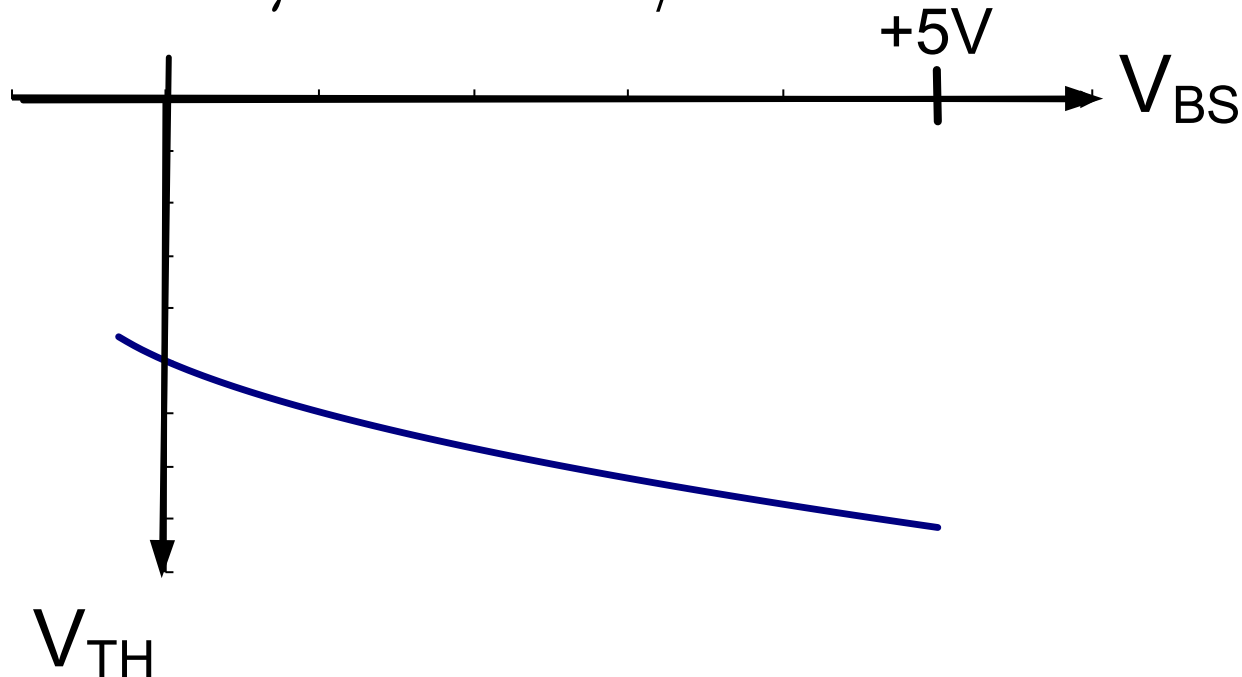
$$\Delta V = V_{TH} - V_{TH0} = \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

$$\Delta V \cong 0.4 \left(\sqrt{0.6V - (-5V)} - \sqrt{0.6} \right) \cong 0.64V$$

Typical Bulk Effects on Threshold Voltage for p-channel Devices

$$V_{TH} = V_{TH0} - \gamma \left(\sqrt{\phi + V_{BS}} - \sqrt{\phi} \right)$$

$$\gamma \cong 0.4V^{1/2} \quad \phi \cong 0.6V$$

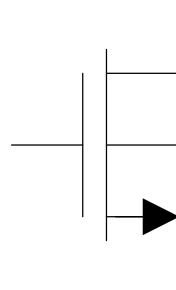


- Bulk-Diffusion Generally Reverse Biased ($V_{BS} > 0$ or at least $V_{BS} > -0.3V$) for p-channel
- Same functional form as for n-channel but $V_{TH0} < 0$
- Magnitude of threshold voltage increases with magnitude of reverse bias

Model Extension Summary

$$I_G = 0$$

$$I_B = 0$$

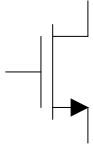


$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

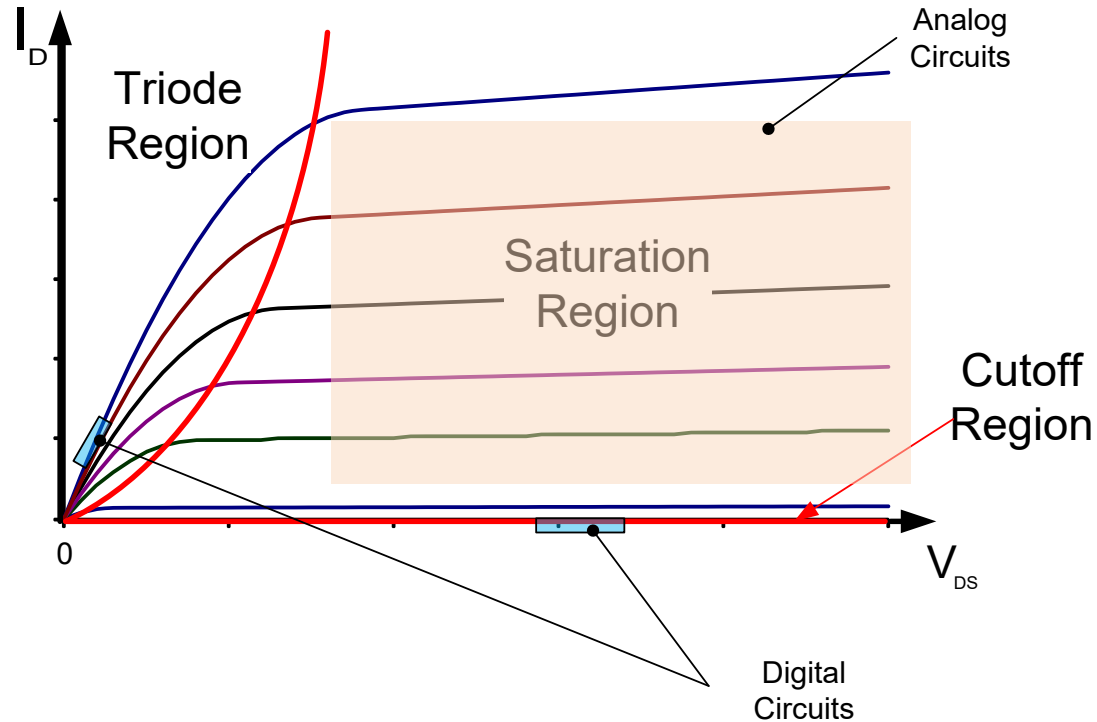
$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}, \phi, \gamma, \lambda\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L



Operation Regions by Applications



Most analog circuits operate in the saturation region

(basic VVR operates in triode and is an exception)

Most digital circuits operate in triode and cutoff regions and switch between these two with Boolean inputs

Model Extension (short devices)

$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

As the channel length becomes very short, velocity saturation will occur in the channel and this will occur with electric fields around $2V/u$. So, if a gate length is around $1u$, then voltages up to $2V$ can be applied without velocity saturation. But, if gate length decreases and voltages are kept high, velocity saturation will occur

$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \frac{\theta_2}{\theta_1} \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\frac{\alpha}{2}} V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \\ \theta_2 \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\alpha} & V_{GS} \geq V_{TH} \quad V_{DS} \geq \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \geq \alpha \geq 1$

Model Extension (short devices)

(n-channel device)

$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \frac{\theta_2}{\theta_1} \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\frac{\alpha}{2}} V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \\ \theta_2 \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\alpha} & V_{GS} \geq V_{TH} \quad V_{DS} \geq \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \geq \alpha \geq 1$

No longer a square-law model (some term it an α -power or α -law model)

For long devices, $\alpha=2$

Channel length modulation (λ) and bulk effects can be added to the velocity Saturation as well

Degrading of α is not an attractive limitation of the MOSFET

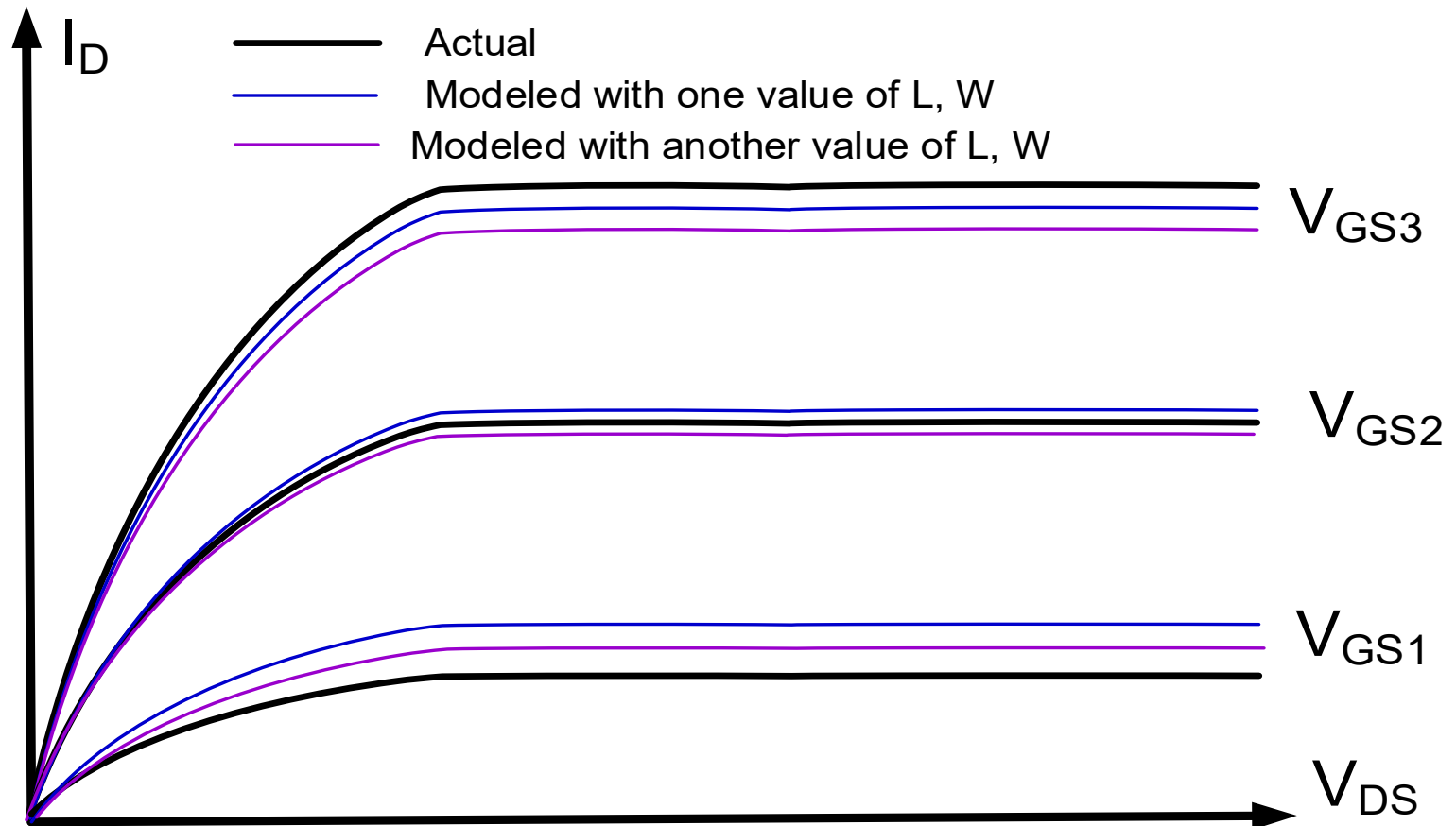
Be aware of existence but of little use !

(too complicated for analytical calculations, not accurate enough for simulations)

Model Extension (BSIM model)

```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM = 27          LEVEL = 49
+XJ = 1.5E-7           NCH = 1.7E17       TOX = 1.42E-8
+K1 = 0.8976376       K2 = -0.09255       VTH0 = 0.629035
+K3B = -8.2369696     W0 = 1.041146E-8    K3 = 24.0984767
+DVT0W = 0            DVT1W = 0           NLX = 1E-9
+DVT0 = 2.7123969     DVT1 = 0.4232931   DVT2W = 0
+U0 = 451.2322004     UA = 3.091785E-13  DVT2 = -0.1403765
+UC = 1.22401E-11     VSAT = 1.715884E5  UB = 1.702517E-18
+AGS = 0.130484       B0 = 2.446405E-6   A0 = 0.6580918
+KETA = -3.043349E-3  A1 = 8.18159E-7    B1 = 5E-6
+RDSW = 1.367055E3    PRWG = 0.0328586   A2 = 0.3363058
+WR = 1               WINT = 2.443677E-7 PRWB = 0.0104806
+XL = 1E-7            XW = 0              LINT = 6.999776E-8
+DWB = 3.676235E-8    VOFF = -1.493503E-4 DWG = -1.256454E-8
+CIT = 0              CDSC = 2.4E-4       NFACTOR = 1.0354201
+CDSCB = 0            ETA0 = 2.342963E-3 CDSCD = 0
+DSUB = 0.0764123     PCLM = 2.5941582   ETAB = -1.5324E-4
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505 PDIBLC1 = 0.8187825
+PSCBE1 = 6.611774E8  PSCBE2 = 3.238266E-4 DROUT = 0.9919348
+PRT = 0              UTE = -1.5          PVAG = 0
+KT1L = 0             KT2 = 0.022         KT1 = -0.11
+UB1 = -7.61E-18     UC1 = -5.6E-11     UA1 = 4.31E-9
+WL = 0               WLN = 1             AT = 3.3E4
+WWN = 1              WWL = 0             WW = 0
+LLN = 1              LWL = 0             LL = 0
+LWL = 0              CAPMOD = 2          LWN = 1
+CGDO = 2.32E-10     CGSO = 2.32E-10    XPART = 0.5
+CJ = 4.282017E-4     PB = 0.9317787     CGBO = 1E-9
+CJSW = 3.034055E-10 PBSW = 0.8          MJ = 0.4495867
+CJSWG = 1.64E-10    PBSWG = 0.8        MJSW = 0.1713852
+CF = 0               PVTH0 = 0.0520855 MJSWG = 0.1713852
+PK2 = -0.0289036    WKETA = -0.0237483 PRDSW = 112.8875816
*                    )
LKETA = 1.728324E-3
```


Model Errors with Different W/L Values



Binning models can improve model accuracy

BSIM Binning Model

- Bin on device sizes
- multiple BSIM models !

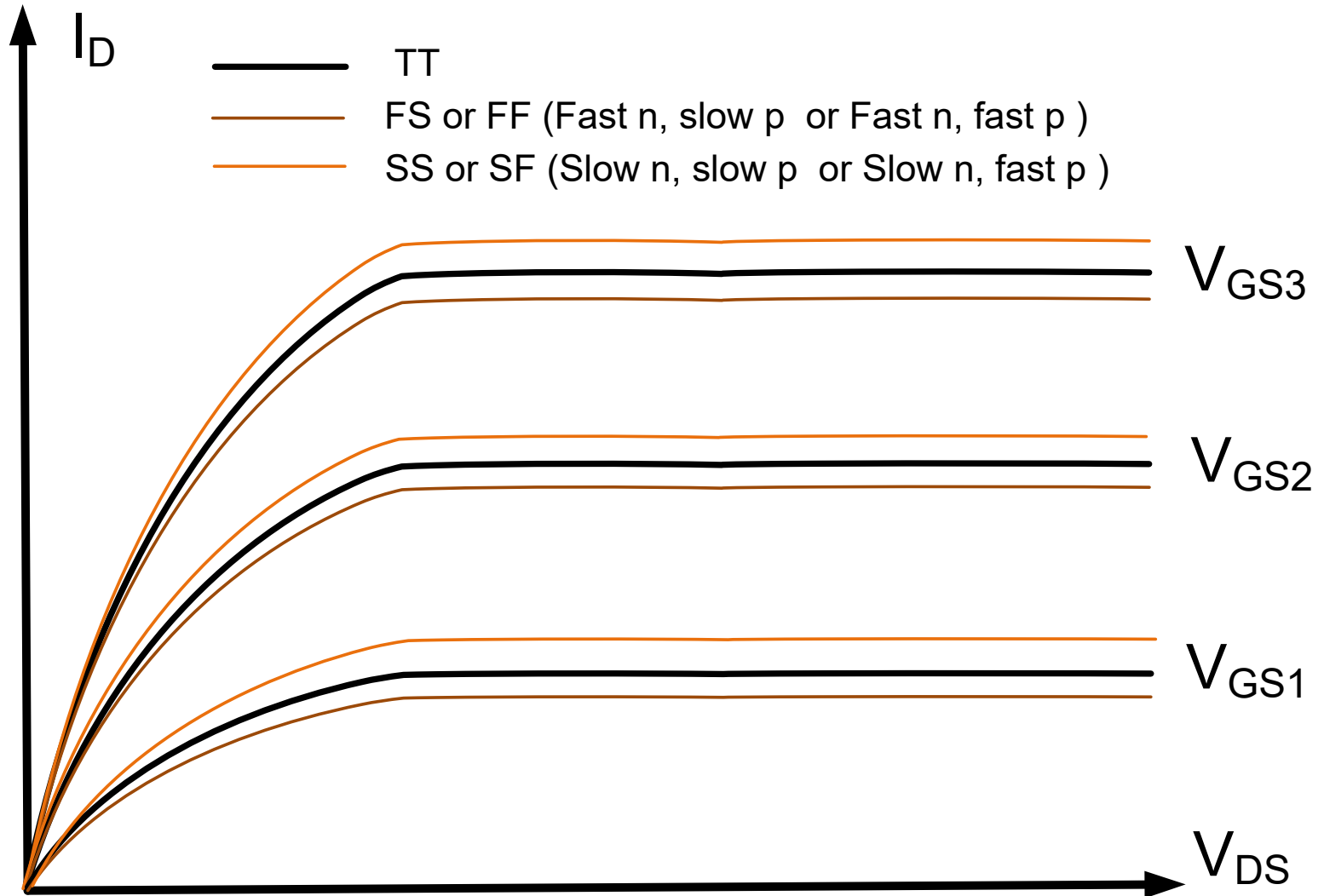
```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM    = 27          TOX    = 1.42E-8
+XJ      = 1.5E-7      NCH     = 1.7E17      VTH0   = 0.629035
+K1      = 0.8976376   K2     = -0.09255     K3     = 24.0984767
+K3B     = -8.2369696  W0     = 1.041146E-8 NLX    = 1E-9
+DVT0W   = 0          DVT1W  = 0          DVT2W  = 0
+DVT0    = 2.7123969  DVT1   = 0.4232931  DVT2   = -0.1403765
+U0      = 451.2322004 UA     = 3.091785E-13 UB     = 1.702517E-18
+UC      = 1.22401E-11 VSAT   = 1.715884E5   A0     = 0.6580918
+AGS     = 0.130484   B0     = 2.446405E-6 B1     = 5E-6
+KETA    = -3.043349E-3 A1     = 8.18159E-7  A2     = 0.3363058
+RDSW    = 1.367055E3 PRWG   = 0.0328586  PRWB   = 0.0104806
+WR      = 1          WINT   = 2.443677E-7 LINT   = 6.999776E-8
+XL      = 1E-7       XW     = 0          DWG    = -1.256454E-8
+DWB     = 3.676235E-8 VOFF   = -1.493503E-4 NFACTOR = 1.0354201
+CIT     = 0          CDSC   = 2.4E-4      CDSCD  = 0
+CDSCB   = 0          ETA0   = 2.342963E-3  ETAB   = -1.5324E-4
+DSUB    = 0.0764123 PCLM   = 2.5941582  PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505   DROUT  = 0.9919348
+PSCBE1  = 6.611774E8 PSCBE2 = 3.238266E-4 PVAG   = 0

+PRT     = 0          UTE    = -1.5       KT1    = -0.11
+KT1L    = 0          KT2    = 0.022      UA1    = 4.31E-9
+UB1     = -7.61E-18 UC1    = -5.6E-11  AT     = 3.3E4
+WL      = 0          WLN    = 1          WW     = 0
+WWN     = 1          WWL    = 0          LL     = 0
+LLN     = 1          LW     = 0          LWN    = 1
+LWL     = 0          CAPMOD = 2          XPART  = 0.5
+CGDO    = 2.32E-10  CGSO   = 2.32E-10  CGBO   = 1E-9
+CJ      = 4.282017E-4 PB     = 0.9317787  MJ     = 0.4495867
+CJSW    = 3.034055E-10 PBSW   = 0.8        MJSW   = 0.1713852
+CJSWG   = 1.64E-10  PBSWG  = 0.8        MJSWG  = 0.1713852
+CF      = 0          PVTH0  = 0.0520855  PRDSW  = 112.8875816
+PK2     = -0.0289036 WKETA  = -0.0237483  LKETA  = 1.728324E-3
*)
*
```

With 32 bins, this model has 3040 model parameters !

Model Changes with Process Variations

(n-ch characteristics shown)



Corner models can improve model accuracy

BSIM Corner Models with Binning

- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS

- bin on device sizes

```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM = 27          TOX = 1.42E-8
+XJ = 1.5E-7           NCH = 1.7E17       VTH0 = 0.629035
+K1 = 0.8976376       K2 = -0.09255       K3 = 24.0984767
+K3B = -8.2369696     W0 = 1.041146E-8     NLX = 1E-9
+DVT0W = 0            DVT1W = 0            DVT2W = 0
+DVT0 = 2.7123969     DVT1 = 0.4232931     DVT2 = -0.1403765
+U0 = 451.2322004     UA = 3.091785E-13    UB = 1.702517E-18
+UC = 1.22401E-11     VSAT = 1.715884E5    A0 = 0.6580918
+AGS = 0.130484       B0 = 2.446405E-6     B1 = 5E-6
+KETA = -3.043349E-3  A1 = 8.18159E-7      A2 = 0.3363058
+RDSW = 1.367055E3    PRWG = 0.0328586     PRWB = 0.0104806
+WR = 1               WINT = 2.443677E-7   LINT = 6.999776E-8
+XL = 1E-7            XW = 0               DWG = -1.256454E-8
+DWB = 3.676235E-8    VOFF = -1.493503E-4  NFACTOR = 1.0354201
+CIT = 0              CDSC = 2.4E-4         CDSCD = 0
+CDSCB = 0            ETA0 = 2.342963E-3   ETAB = -1.5324E-4
+DSUB = 0.0764123     PCIM = 2.5941582     PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505 DROUT = 0.9919348
+PSCBE1 = 6.611774E8  PSCBE2 = 3.238266E-4 PVAG = 0
+DPR.TA = 0.01        RSH = 83.5           MORMOD = 1
+PRT = 0              UTE = -1.5           KT1 = -0.11
+KT1L = 0             KT2 = 0.022          UA1 = 4.31E-9
+UB1 = -7.61E-18      UC1 = -5.6E-11       AT = 3.3E4
+WL = 0               WLN = 1              WW = 0
+WWN = 1              WWL = 0              LL = 0
+LLN = 1              LW = 0               LWN = 1
+LWL = 0              CAPMOD = 2           XPART = 0.5
+CGDO = 2.32E-10      CGSO = 2.32E-10      CGBO = 1E-9
+CJ = 4.282017E-4     PB = 0.9317787       MJ = 0.4495867
+CJSW = 3.034055E-10  PBSW = 0.8           MJSW = 0.1713852
+CJSWG = 1.64E-10     PBSWG = 0.8          MJSWG = 0.1713852
+CF = 0               PVTH0 = 0.0520855    PRDSW = 112.8875816
+PK2 = -0.0289036     WKETA = -0.0237483   LKETA = 1.728324E-3 )
*
```

With 32 size bins and 4 corners, this model has 15,200 model parameters !

How many models of the MOSFET do we have?

Switch-level model (2)

Square-law model

Square-law model (with λ and bulk additions)

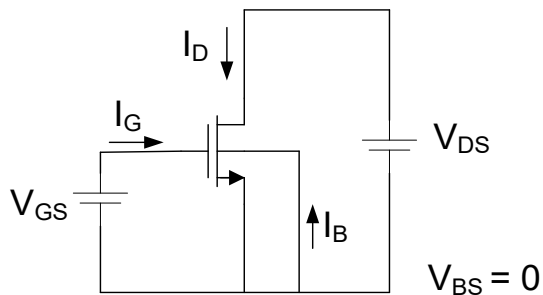
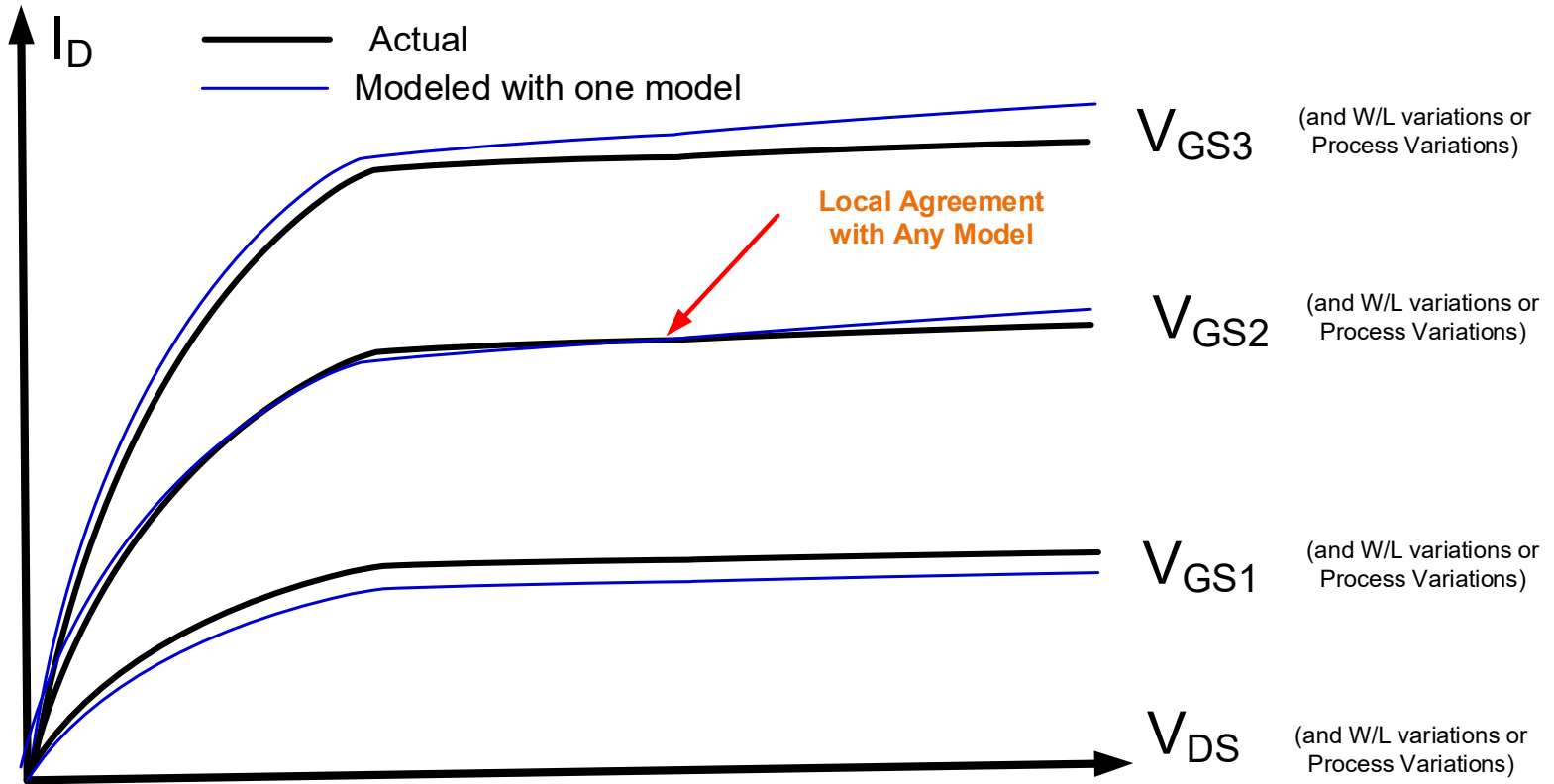
α -law model (with λ and bulk additions)

BSIM model

BSIM model (with binning extensions)

BSIM model (with binning extensions and process corners)

The Modeling Challenge



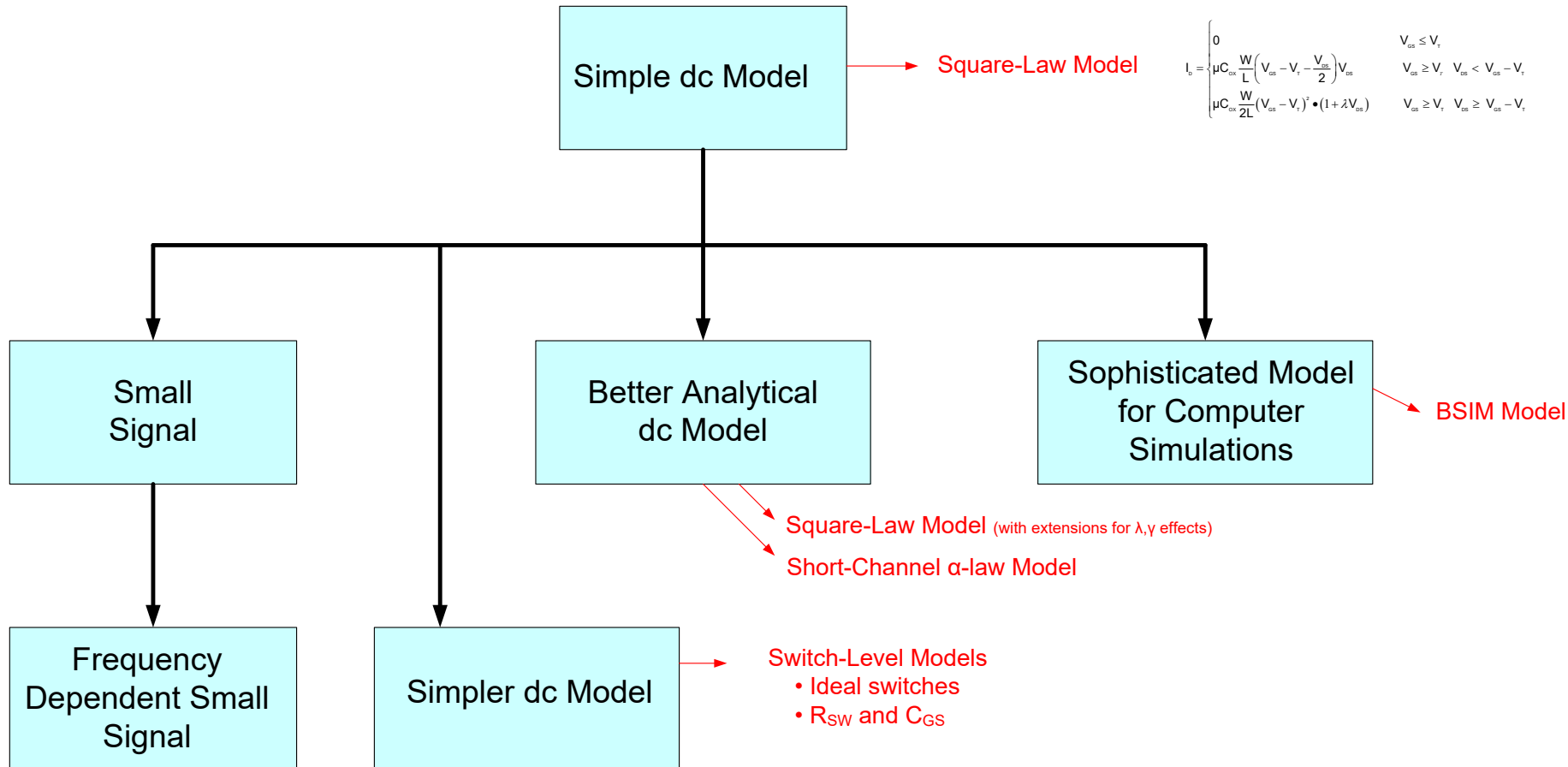
$$I_D = f_1(V_{GS}, V_{DS})$$

$$I_G = f_2(V_{GS}, V_{DS})$$

$$I_B = f_3(V_{GS}, V_{DS})$$

Difficult to obtain analytical functions that accurately fit actual devices over bias, size, and process variations

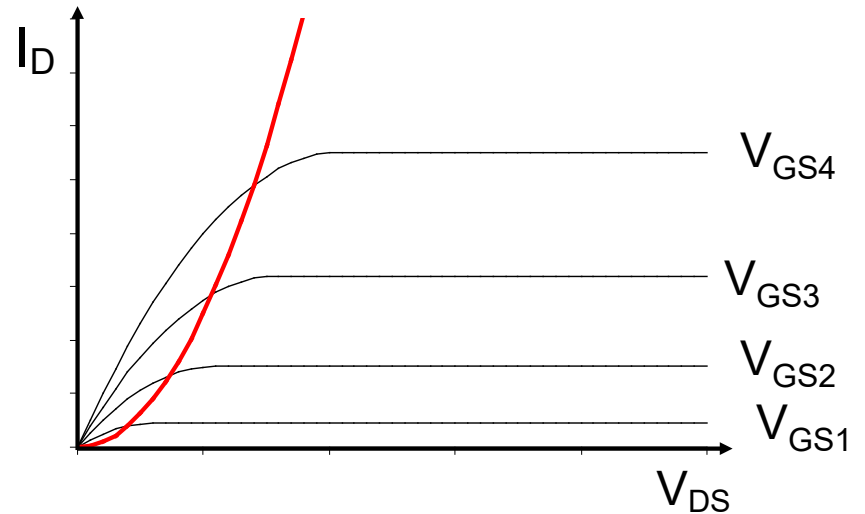
Model Status



In the next few slides, the models we have developed will be listed and reviewed

- Square-law Model
- Switch-level Models
- Extended Square-law model
- Short-channel model
- BSIM Model
- BSIM Binning Model
- Corner Models

Square-Law Model

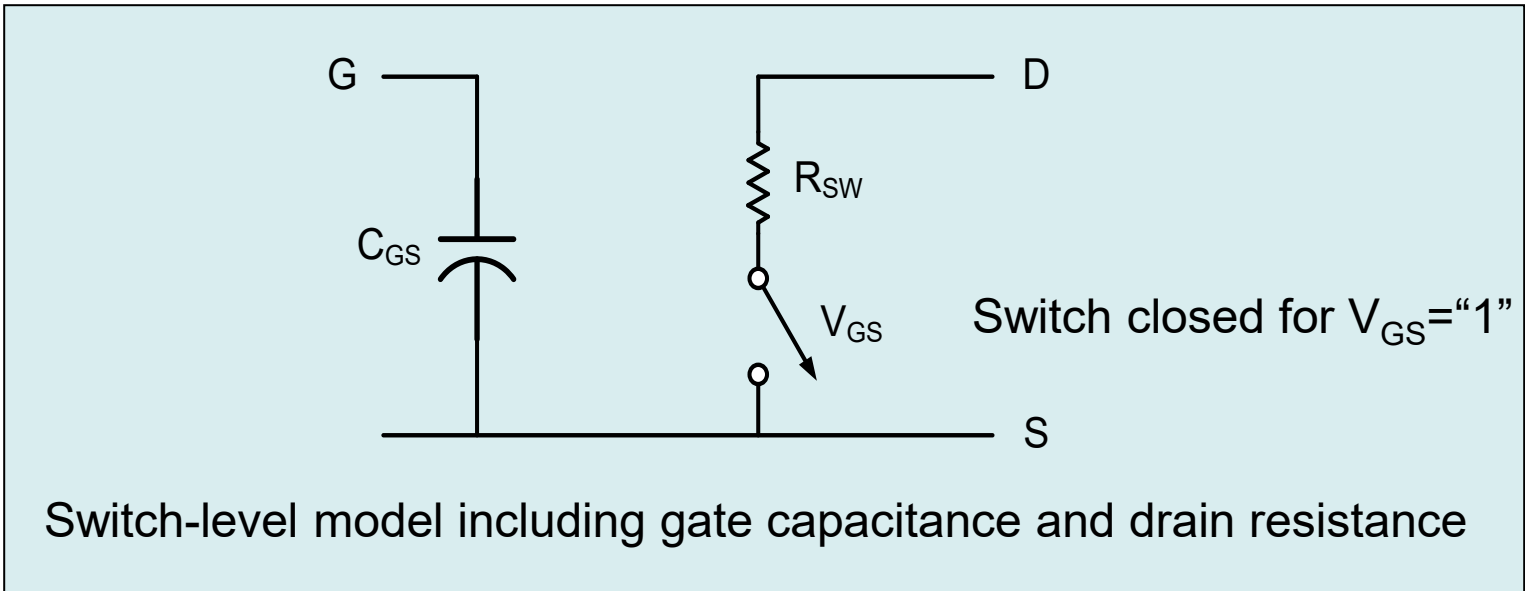
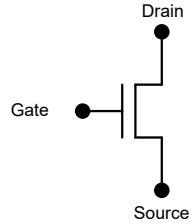


$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L

Switch-Level Models



C_{GS} and R_{SW} dependent upon device sizes and process

For minimum-sized devices in a 0.5u process

$$\mathbf{C_{GS} \cong 1.5fF} \quad \mathbf{R_{sw} \cong \left. \begin{array}{l} 2K\Omega \text{ n-channel} \\ 6K\Omega \text{ p-channel} \end{array} \right\}}$$

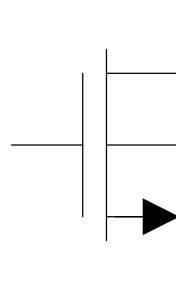
Considerable emphasis will be placed upon device sizing to manage C_{GS} and R_{SW}

Model Parameters : $\{C_{GS}, R_{SW}\}$

Extended Square-Law Model

$$I_G = 0$$

$$I_B = 0$$



$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_{TH} \quad V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}, \phi, \gamma, \lambda\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L

Short-Channel Model

$$I_D = \begin{cases} 0 & V_{GS} \leq V_{TH} \\ \frac{\theta_2}{\theta_1} \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\frac{\alpha}{2}} V_{DS} & V_{GS} \geq V_{TH} \quad V_{DS} < \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \\ \theta_2 \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH})^{\alpha} & V_{GS} \geq V_{TH} \quad V_{DS} \geq \theta_1 (V_{GS} - V_{TH})^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \geq \alpha \geq 1$

Channel length modulation (λ) and bulk effects can be added to the velocity Saturation as well

BSIM model

```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM    = 27          LEVEL  = 49
+XJ      = 1.5E-7      NCH     = 1.7E17      TOX    = 1.42E-8
+K1      = 0.8976376   K2     = -0.09255    VTH0   = 0.629035
+K3B     = -8.2369696  W0     = 1.041146E-8 K3     = 24.0984767
+DVT0W   = 0          DVT1W  = 0          NLX    = 1E-9
+DVT0    = 2.7123969  DVT1   = 0.4232931  DVT2W  = 0
+U0      = 451.2322004 UA     = 3.091785E-13 DVT2   = -0.1403765
+UC      = 1.22401E-11 VSAT   = 1.715884E5   UB     = 1.702517E-18
+AGS     = 0.130484   B0     = 2.446405E-6 A0     = 0.6580918
+KETA    = -3.043349E-3 A1     = 8.18159E-7  B1     = 5E-6
+RDSW    = 1.367055E3 PRWG   = 0.0328586  A2     = 0.3363058
+WR      = 1          WINT   = 2.443677E-7 PRWB   = 0.0104806
+XL      = 1E-7      XW     = 0          LINT   = 6.999776E-8
+DWB     = 3.676235E-8 VOFF   = -1.493503E-4 DWG    = -1.256454E-8
+CIT     = 0          CDSC   = 2.4E-4      NFACTOR = 1.0354201
+CDSCB   = 0          ETA0   = 2.342963E-3 CDSCD  = 0
+DSUB    = 0.0764123 PCLM   = 2.5941582  ETAB   = -1.5324E-4
+PDIBLC2 = 2.366707E-3 PCLM   = 2.5941582  PDIBLC1 = 0.8187825
+PSCBE1  = 6.611774E8 PSCBE2 = -0.0431505 DROUT  = 0.9919348
+PRT     = 0          UTE    = -1.5      PVAG   = 0
+KT1L    = 0          KT2    = 0.022     KT1    = -0.11
+UB1     = -7.61E-18 UC1     = -5.6E-11 UA1    = 4.31E-9
+WL      = 0          WLN    = 1        AT     = 3.3E4
+WVN     = 1          WWL    = 0        WW     = 0
+LLN     = 1          LW     = 0        LL    = 0
+LWL     = 0          CAPMOD = 2        LWN   = 1
+CGDO    = 2.32E-10  CGSO   = 2.32E-10  XPART  = 0.5
+CJ      = 4.282017E-4 PB      = 0.9317787 CGBO   = 1E-9
+CJSW    = 3.034055E-10 PBSW   = 0.8       MJ     = 0.4495867
+CJSWG   = 1.64E-10  PBSWG  = 0.8       MJSW  = 0.1713852
+CF      = 0          PVTH0  = 0.0520855 MJSWG = 0.1713852
+PK2     = -0.0289036 WKETA  = -0.0237483 PRDSW  = 112.8875816
*                                     LKETA  = 1.728324E-3 )
```

Note this model has 95 model parameters !

BSIM Binning Model

- Bin on device sizes
- multiple BSIM models !

```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM    = 27          TOX      = 1.42E-8
+XJ      = 1.5E-7      NCH     = 1.7E17       VTH0    = 0.629035
+K1      = 0.8976376  K2     = -0.09255      K3       = 24.0984767
+K3B     = -8.2369696 W0     = 1.041146E-8  NLX      = 1E-9
+DVT0W   = 0          DVT1W  = 0           DVT2W   = 0
+DVT0    = 2.7123969  DVT1   = 0.4232931   DVT2    = -0.1403765
+U0      = 451.2322004 UA     = 3.091785E-13  UB       = 1.702517E-18
+UC      = 1.22401E-11 VSAT   = 1.715884E5   A0       = 0.6580918
+AGS     = 0.130484   B0     = 2.446405E-6  B1       = 5E-6
+KETA    = -3.043349E-3 A1     = 8.18159E-7   A2       = 0.3363058
+RDSW    = 1.367055E3 PRWG   = 0.0328586   PRWB     = 0.0104806
+WR      = 1          WINT   = 2.443677E-7  LINT     = 6.999776E-8
+XL      = 1E-7      XW     = 0           DWG      = -1.256454E-8
+DWB     = 3.676235E-8 VOFF   = -1.493503E-4 NFACTOR  = 1.0354201
+CIT     = 0          CDSC   = 2.4E-4       CDSCD    = 0
+CDSCB   = 0          ETA0   = 2.342963E-3   ETAB     = -1.5324E-4
+DSUB    = 0.0764123 PCLM   = 2.5941582   PDIBLC1  = 0.8187825
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505    DROUT    = 0.9919348
+PSCBE1  = 6.611774E8 PSCBE2 = 3.238266E-4  PVAG     = 0

+PRT     = 0          UTE    = -1.5        KT1      = -0.11
+KT1L    = 0          KT2    = 0.022       UA1      = 4.31E-9
+UB1     = -7.61E-18 UC1     = -5.6E-11    AT       = 3.3E4
+WL      = 0          WLN    = 1          WW       = 0
+WWN     = 1          WWL    = 0          LL       = 0
+LLN     = 1          LW     = 0          LWN     = 1
+LWL     = 0          CAPMOD = 2          XPART    = 0.5
+CGDO    = 2.32E-10  CGSO   = 2.32E-10   CGBO     = 1E-9
+CJ      = 4.282017E-4 PB      = 0.9317787   MJ       = 0.4495867
+CJSW    = 3.034055E-10 PBSW   = 0.8         MJSW    = 0.1713852
+CJSWG   = 1.64E-10  PBSWG  = 0.8         MJSWG   = 0.1713852
+CF      = 0          PVTH0  = 0.0520855   PRDSW   = 112.8875816
+PK2     = -0.0289036 WKETA  = -0.0237483   LKETA   = 1.728324E-3 )
*
```

With 32 bins, this model has 3040 model parameters !

BSIM Corner Models

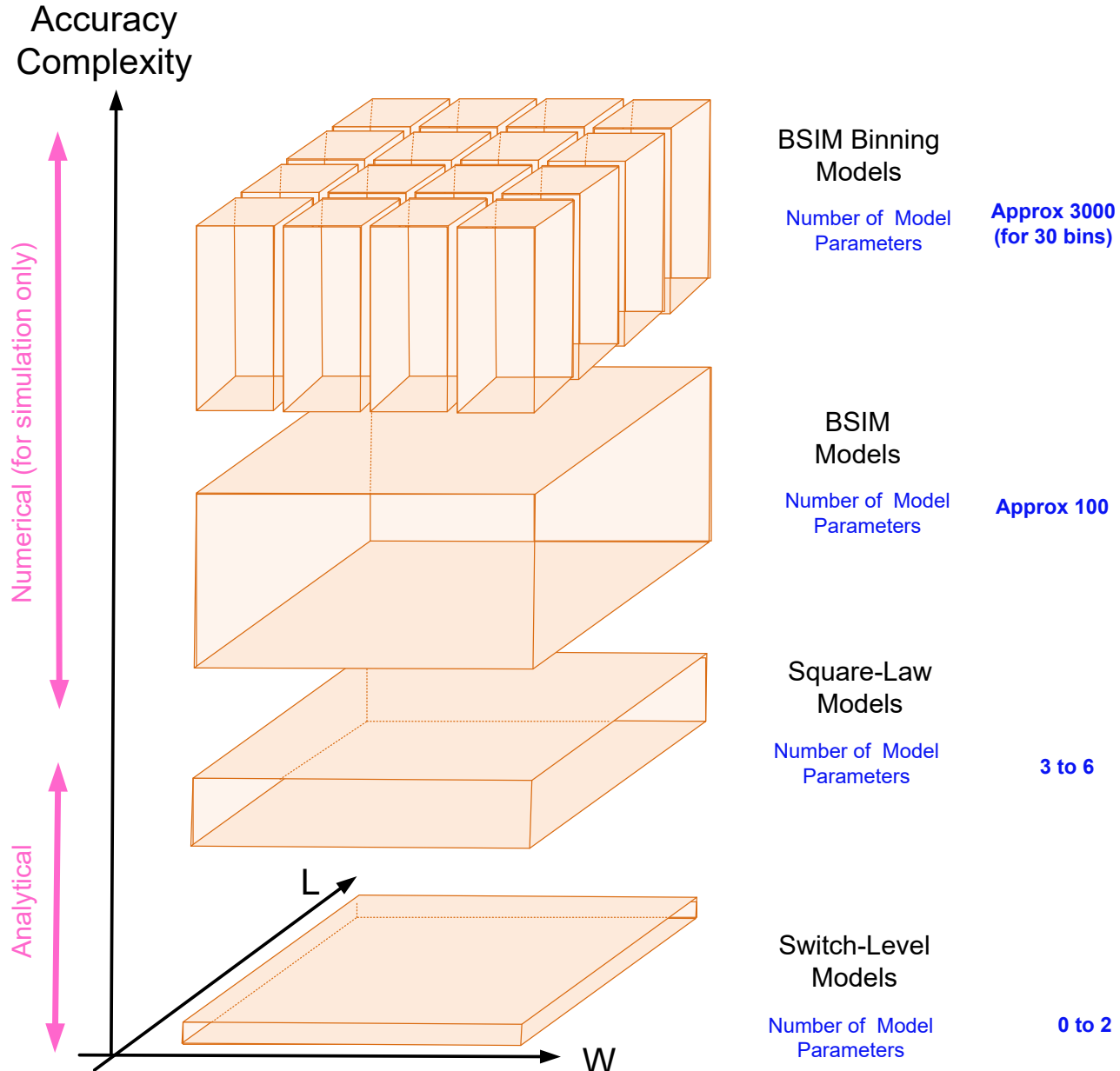
- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS
 - five different BSIM models !

```
.MODEL CMOSN NMOS (
+VERSION = 3.1          TNOM    = 27          LEVEL   = 49
+XJ      = 1.5E-7       NCH     = 1.7E17       TOX     = 1.42E-8
+K1      = 0.8976376    K2     = -0.09255      VTH0    = 0.629035
+K3B     = -8.2369696   W0     = 1.041146E-8  K3      = 24.0984767
+DVT0W   = 0           DVT1W  = 0           NLX     = 1E-9
+DVT0    = 2.7123969   DVT1   = 0.4232931   DVT2W   = 0
+U0      = 451.2322004  UA     = 3.091785E-13  DVT2    = -0.1403765
+UC      = 1.22401E-11 VSAT    = 1.715884E5    UB      = 1.702517E-18
+AGS     = 0.130484    B0     = 2.446405E-6   A0      = 0.6580918
+KETA    = -3.043349E-3 A1     = 8.18159E-7    B1      = 5E-6
+RDSW    = 1.367055E3  PRWG   = 0.0328586    A2      = 0.3363058
+WR      = 1           WINT   = 2.443677E-7  PRWB   = 0.0104806
+XL      = 1E-7       XW     = 0           LINT    = 6.999776E-8
+DWB     = 3.676235E-8 VOFF   = -1.493503E-4 DWG     = -1.256454E-8
+CIT     = 0           CDSC   = 2.4E-4        NFACTOR = 1.0354201
+CDSCB   = 0          ETA0   = 2.342963E-3   CDSCD   = 0
+DSUB    = 0.0764123  PCLM   = 2.5941582   ETAB    = -1.5324E-4
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505    PDIBLC1 = 0.8187825
+PSCBE1  = 6.611774E8 PSCBE2 = 3.238266E-4  DROUT   = 0.9919348
+DRIFTA  = 0.01       RSH    = 83.5        PVAG    = 0
+PRT     = 0           UTE    = -1.5        MORMOD  = 1
+KT1L    = 0           KT2    = 0.022       KT1     = -0.11
+UB1     = -7.61E-18  UC1    = -5.6E-11    UA1     = 4.31E-9
+WL      = 0           WLN    = 1           AT      = 3.3E4
+WWN     = 1           WWL    = 0           WW      = 0
+LLN     = 1           LW     = 0           LL      = 0
+LWL     = 0           CAPMOD = 2           LWN     = 1
+CGDO    = 2.32E-10   CGSO   = 2.32E-10    XPART   = 0.5
+CJ      = 4.282017E-4 PB      = 0.9317787   CGBO    = 1E-9
+CJSW    = 3.034055E-10 PBSW   = 0.8           MJ      = 0.4495867
+CJSWG   = 1.64E-10  PBSWG  = 0.8           MJSW   = 0.1713852
+CF      = 0           PVTH0  = 0.0520855    MJSWG  = 0.1713852
+PK2     = -0.0289036 WKETA  = -0.0237483   PRDSW   = 112.8875816
                                         LKETA  = 1.728324E-3 )
*
```

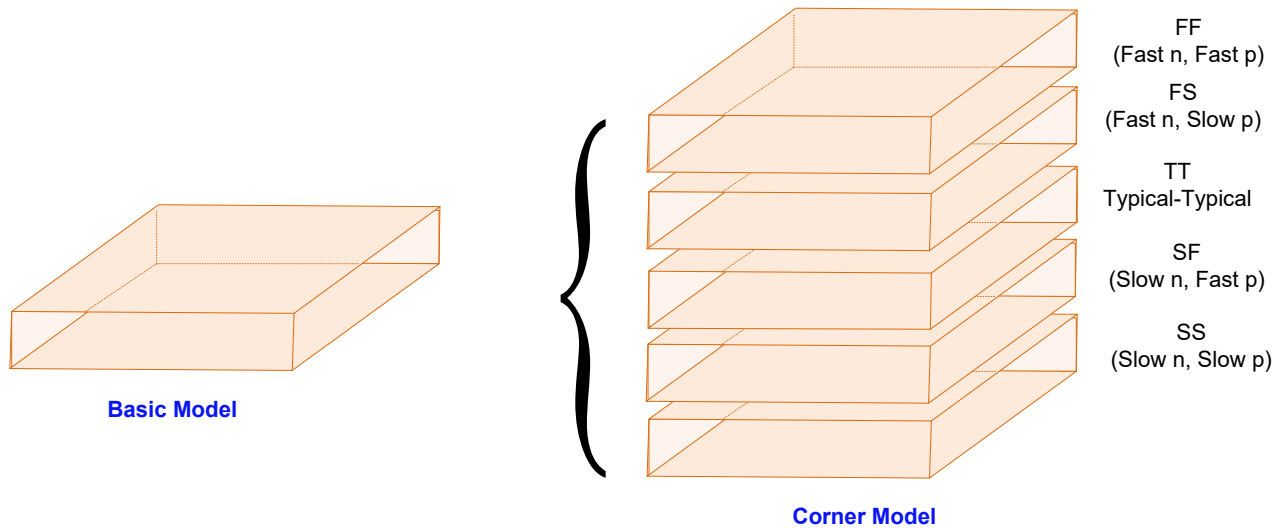
TT: typical-typical
FF: fast n, fast p
FS: fast n, slow p
SF: slow n, fast p
SS: slow n, slow p

With 4 corners, this model has 475 model parameters !

Hierarchical Model Comparisons



Corner Models



Applicable at any level in model hierarchy (same model, different parameters)

Often 4 corners (FF, FS, SF, SS) used but sometimes many more

Designers must provide enough robustness so good yield at all corners



Stay Safe and Stay Healthy !

End of Lecture 17